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Preliminary Evaluation of Sulfur Variability in Low-Sulfur Coals from Selected Mines

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PRELIMINARY EVALUATION OF SULFUR VARIABILITY IN LOW-SULFUR COALS FROM SELECTED MINES

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16. ABSTRACT

Data on the variability of sulfur content and heating value of coal have been obtained from several coal and utility companies. These data have been analyzed to estimate the mean, standard deviation, and the frequency distribution of weight percent sulfur (dry basis) and the impact of this variability on the required average sulfur content to comply with an emission regulation.

In order to assess the implications of sulfur variability for compliance with emission regulations, such as those specified by State Implementation Plans(SIP) and New Source Performance Standards (NSPS), the relative standard deviation (RSD) of weight percent sulfur was estimated as a function of the amount of coal sampled and the number of composite samples for the specified averaging period. These RSD's were used to estimate the required average sulfur content to yield 95 and 99 percent compliance with the emission limitation of 1.2 lb SO_2/MM Btu. The detailed computations are given for two sizes of power plants, 500 MW and 25 MW (250 MM Btu/hour, the smallest plant covered by the NSPS). As the percent of compliance increases the required average sulfur content decreases and rapidly approaches sulfur levels for which the availability of uncleaned coal to comply with the NSPS would apporach zero.

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SUMMARY

Data on the variability of sulfur content and heating value of coal were obtained from several coal and utility companies. These data were analyzed to estimate the mean, standard deviation, and the frequency distribution of weight percent sulfur (dry basis) and the impact of this variability on the average sulfur content required for compliance with an emission regulation. Analysis of composite samples of coal from unit trains indicates that the values of weight percent sulfur are skewed to the right--that is, they tail off slowly at the higher sulfur levels. This finding suggests the use of a skewed distribution such as the logarithmic normal or the inverted gamma distribution[1] in empirical approximations of the frequency distribution of the weight percent sulfur. The data on heating values (Btu/lb) do not appear to be skewed. The ratio of sulfur dioxide emitted per million Btu (lb SO_2/MM Btu) is also skewed because of the dominating influence of the variation of sulfur content. This report indicates that the relative standard deviation (RSD, the ratio of the standard deviation to the mean expressed as a percent) for sulfur content expressed as lb SO_2/MM Btu is 1.02 to 1.05 times the RSD for weight percent sulfur. The statistical correlation between sulfur content and heating value was found to be relatively low but significant in a few cases, and insignificantly different from zero for almost all of the data sets used in this study.

In order to assess the implications of sulfur variability in coal for compliance with emission regulations such as those specified in State Implementation Plans (SIP) and New Source Performance Standards (NSPS), the relative standard deviation of weight percent sulfur was estimated

as a function of the amount of coal sampled and the number of composite samples for the specified averaging period, e.g. 1 week, 1 month.

These RSD's were then used to estimate the average sulfur content required to yield 95 and 99 percent compliance with the NSPS emission limitation of 1.2 lb SO_2/MM Btu. Detailed computations are given for two sizes of power plants, 500 MW and 25 MW* (250 MM Btu/hour, the smallest plant covered by the NSPS). Based on averaging periods of 1 month (3 hours), the required averages of weight percent sulfur to achieve 95 percent compliance are estimated to be 0.65 and 0.60 (0.52 and 0.49) for the 500-MW and 25-MW plant, respectively. As the percent of compliance increases, the average sulfur content required for compliance decreases and rapidly approaches sulfur levels at which the availability of uncleaned coal that would allow the user to comply with the NSPS would approach zero.

A methodology is provided whereby an analyst may substitute values of sulfur content variability expressed as an RSD into a computation or use a graph to estimate the average sulfur content, in lb SO_2/MM Btu or weight percent, that is required for compliance with NSPS. Several hypothetical examples are given to illustrate application of the data and methodology. If an analyst cannot obtain data specific to a coal supply, he may use typical or nominal values given in this report. These values may be adjusted upward or downward if the variation would be expected to be greater or less than that presented in the summary tables and graphs of this report.

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*Assumes 10,000 Btu/kWh heat rate

PRELIMINARY EVALUATION OF SULFUR VARIABILITY IN LOW-SULFUR COALS FROM SELECTED MINES

1.0 INTRODUCTION

The primary objectives of this report are (1) to summarize results of a study on sulfur variability in coals and (2) to determine the effect of this variability on compliance with Federal emission regulations. Data were obtained from coal companies, utility companies, EPA files, and earlier reports. The data do not permit a study of sulfur variability for coals from all mines or seams within a district or region, because the data do not represent a random sample of coals from a geographical area. A background discussion concerning variability of sulfur content is given in Section 2, sources of the data used in this study are reported in Section 3, summary results are presented in Section 4, and implications of the results with respect to compliance with emission regulations are discussed in Section 5. Because the volume of data used in the study is too large to include in the report, selected data summaries are given in Appendix A. These data were analyzed by standard statistical methods, described in Appendix B. A possible application of a quality control technique is described in Appendix B.4. Appendix C presents an analysis of the variability of sulfur content as a function of the tonnage of coal sampled. Appendix D gives some comments of reviewers with respect to practical applications of the methods presented herein.

One of the primary objectives of this study is to assess the effect of the variability of sulfur content of coals on compliance with the Federal New Source Performance Standard (NSPS) limiting the emissions of sulfur dioxide from coal-fired steam-generating units. The pertinent portion of this standard follows:

> The provisions of this subpart are applicable to each fossil fuel-fired steam generating unit of more than 63 million kcal per hour heat input (250 million Btu per hour), which is the affected facility. Any change to an existing fossil fuel-fired steam generating unit to accomodate the use of combustible materials, other than fossil fuel as defined in this subpart, shall not bring that unit under the applicability of this subpart.
>
> On and after the date on which the performance test required to be conducted by Part 60.8 of the Federal Register is completed, no owner or operator subject to the provisions of this subpart shall cause to be discharged into the atmosphere from any affected facility any gases which contain sulfur dioxide in excess of 2.2 g per million cal heat input (1.2 lb per million Btu) derived from solid fossil fuel.*

The essence of this report consists of the methodology presented in Section 5 and use of the tables and graphs of that section in analysis of the effect of sulfur variability on compliance with emission regulations. Several examples are given to demonstrate application of the results. The user is in no way restricted to the data given in this report. To the extent possible, the user should obtain data on variability of sulfur content of specific coal supplies under consideration; the user can then apply the appropriate methodology in Section 5.

The scope of this study does not include determination of how the emission regulations will affect the sale and utilization of coals; the attempt here is to provide the information needed for such a determination.

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*Code of Federal Regulations 40, Protection of the Environment, Parts 60, §60.40 and 60.43 (a) (2), (July 1, 1975, page 743.)

2.0 VARIABILITY OF SULFUR IN COAL

2.1 BACKGROUND

The sulfur content of coal occurs as the mineral pyrite, as organically sound sulfur, and in trace amounts as sulfate sulfur. The ratio of organically bound sulfur to inorganic sulfur varies widely from coal seam to coal seam, from mine to mine operating in the same seam and to a lesser extent within the same mining complex.

An example of the seam variability is shown in the data displayed in Figure 2-1 for Pittsburgh seam (West Virginia) coal. The range of organic sulfur is between 1.0 and 2.92 percent by weight, with a mean and standard deviation of 1.62 and 0.52 percent, respectively. The pyritic sulfur content of these same coals ranges from 0.87 to 4.69 percent, with a mean and standard deviation of 2.07 and 0.95 percent, respectively.

In midwestern coals, the organic sulfur content is higher on the average than in eastern coals; however, the organic sulfur occurs in a relatively narrow range of concentrations as compared with the pyritic sulfur in these coals. In most western coals the pyritic sulfur content is extremely low, and therefore little or no reduction in the total sulfur content is achieved by present economical cleaning processes; although in some cases the reduction in sulfur content of western coals would be sufficient to upgrade the coal to NSPS.

In a specific coal reserve, the distribution of total sulfur may occur as shown hypothetically in Figure 2-2, which identifies iso-sulfur contours. The range of sulfur content shown is from 1 to 5 percent. If this reserve were mined by surface or strip mining, the variability of sulfur content from month to month or year to year would

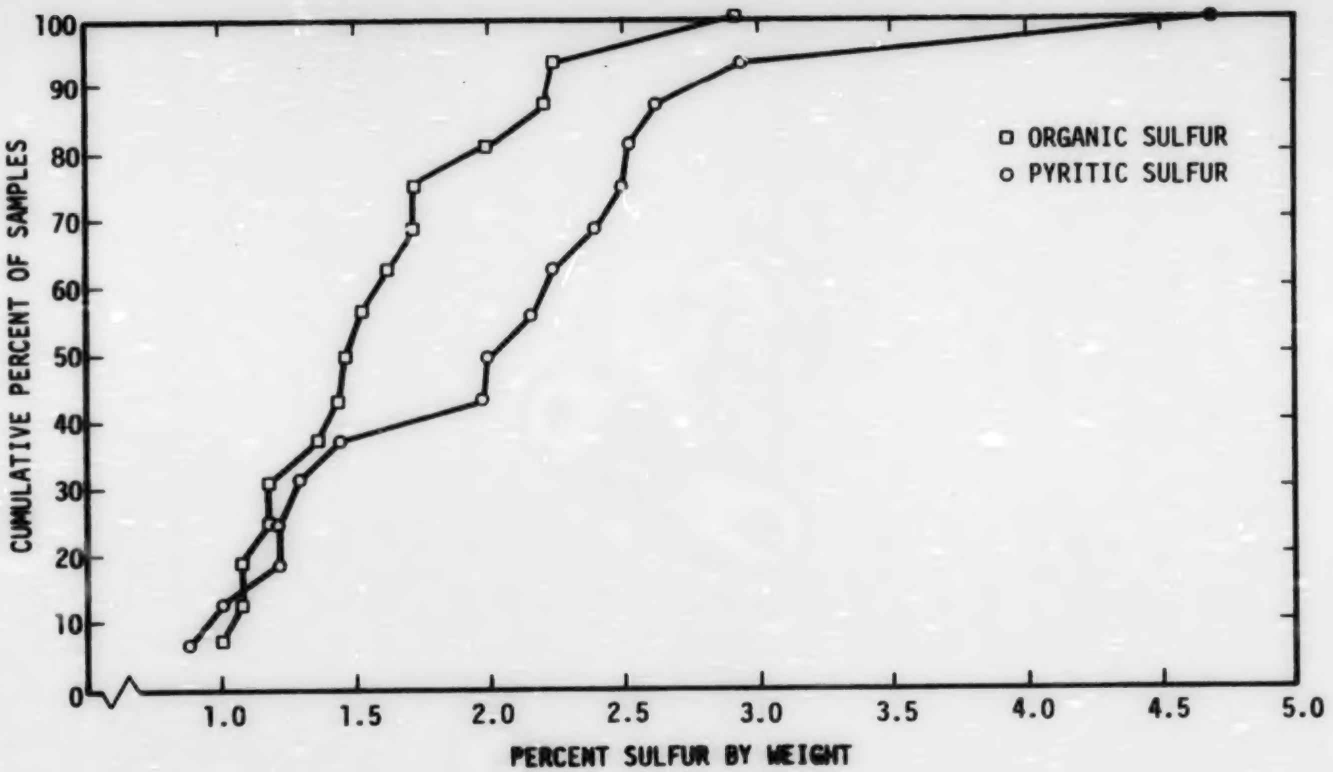

Figure 2-1. Variation in pyritic and organic sulfur contents of Pittsburgh seam coal (West Virginia).

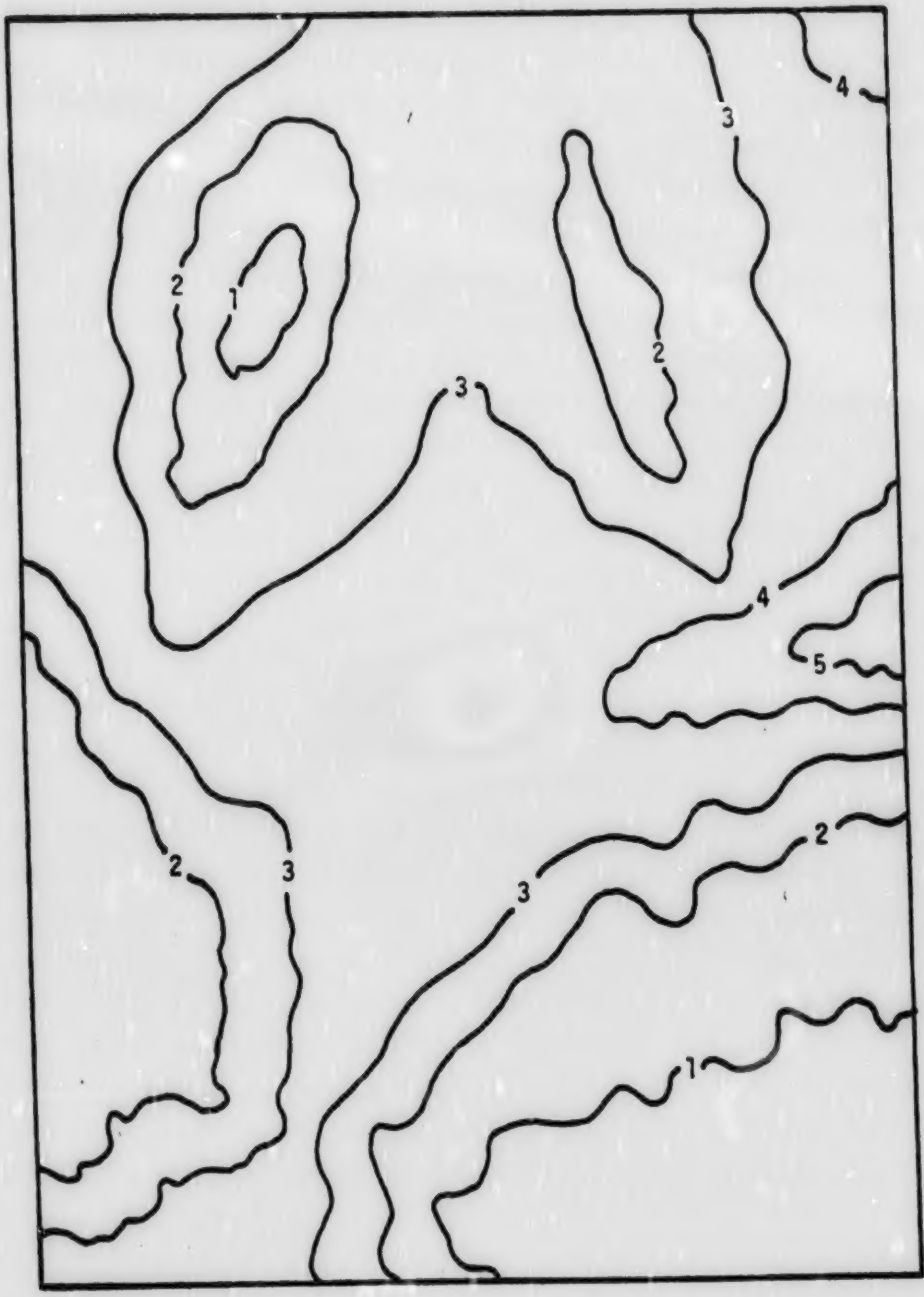
4
3
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Figure 2-2. Hypothetical distribution of percent sulfur in 30-year coal reserve.

be significant, since a single large stripping machine is usually operated along a designated path. In deep mining, however, a number of mining machines operate in different sections of the reserve, generally advancing at a constant rate. Variability of sulfur content of the mined product on a daily basis would be higher than the variability of the average sulfur content on a monthly or annual basis. The daily variability would reflect the in-seam variability, the number of machines operating, the type of deep mining and the relative productivity of these machines.

Study of sulfur content of coals requires consideration of a number of variables such as those summarized in Table 2-1. Many of these factors are interrelated. For example, the measured differences in sulfur content can reflect not only differences in the coal in the mine or differences in the mining method, but also differences in the methods of sampling, averaging, and measurement. Variability of sulfur content can be determined on the basis of (1) individual samples, e.g., one composite sample taken from each unit train (60 to 100 cars or 6,000 to 10,000 tons) by the Standard ASTM Method[3] and/or (2) weighted averages of these individual samples, corresponding to weekly or monthly averages of coal as received at a utility plant. The four coal companies providing coal data for this study use this ASTM method. The data on sulfur variability can then be used in determining the required average sulfur content of the coal for the user to comply with a given regulation.

Table 2-1 PERTINENT FACTORS IN STUDYING SULFUR VARIABILITY

1. Type of coal

   Organic and inorganic sulfur content.
   Distribution of sulfur in coal (coarse pyrite or finely disseminated throughout the coal). The form in which the sulfur occurs is significant when the coal is washed.

2. Stage of sampling

   Core drilling (or channel samples after operation).
   Run-of-mine production.
   After preparation, cleaning.
   As received at utility plant/consumer.
   As burned.

3. Coal blending and processing

4. Mining plan (selective)

5. Mining technique

   Number and location of machines, type of mining

6. Location of coal

   Seam.
   Mine.
   Region or district.

7. Averaging times/tonnages

   Daily.
   Weekly.
   Monthly.
   Other.

8. Sampling procedure

   Amount of coal sampled.
   How the sample is formed as a composite of a large amount of coal.
   Sample variation.

9. Analytical method

10. Cleaning technique

Variability in sulfur content and heating value of coals could be associated with each of the factors listed in Table 2-1. A detailed analysis would not be practical, however, in view of the large number of factors and their interdependence. Even though some of this information is available in the records of operating mines and steam plants, the cost of assembling and analyzing the data would be prohibitive.

2.2 APPROACH

The approach used in this study was to obtain data on sulfur contents and heating values of coal from several sources and to analyze these data by standard statistical techniques to obtain the following summary information:

1. Averages of sulfur content (weight percent sulfur), heating value (Btu/lb), and sulfur emissions (lb SO_2/MM Btu) over different averaging times (per unit train, weekly, monthly, etc.).
2. Standard deviations of weight percent sulfur, Btu/lb, and lb SO_2/MM Btu.
    a. Among individual samples from unit trains.
    b. Within and among weekly averages.
    c. Within and among monthly averages.
3. Frequency distribution of weight percent sulfur, Btu/lb, and lb SO_2/MM Btu
    a. Histograms of data.
    b. Cumulative frequency distribution.

Depending upon the particular data set, one or more of these analyses was conducted for each summary statistic.

No attempt was made to obtain random samples of coals from different mines and seams in a given region or district, even though this approach was considered early in the study.

Because data sources do not coincide with a region or district, inferences cannot be made with respect to the overall variability of all coals in a sampled region. The results are nonetheless useful in assessing the impact of variability of sulfur content on compliance with emission regulations.

It would be desirable to know the long-term variability of sulfur content because compliance with emission control regulations is required for the life of the plant. None of the data, however, extend over a long enough period to permit estimation of long-term variability.

No attempt was made to isolate the variation in measured sulfur content that is due to the analytical measurement. With respect to coals containing less than 2 weight percent sulfur, one study indicates that the analytical reproducibility (the differences between two or more determinations carried out by different laboratories on representative samples taken from the same bulk sample) should be within 0.10 percent in absolute terms.[4] For example, if the mean sulfur content were 0.70 weight percent, the values measured by different laboratories should fall within an interval of 0.65 to 0.75 weight percent. Reference 4 and Appendix B.1.3 provide further details of the analytical method and its precision.

3.0 DATA BASE

The data base is comprised of 35 data sets. Table 3-1 lists the data sets with comments on each; the sets are numbered serially with a prefix U or C to denote utility or coal company as the source. All of the sets represent coals mined in recent years (1974, 1975 and 1976), and the sulfur measurements reflect either core samples or composite samples from unit trains. A composite sample may represent 1,000 to 20,000 tons of coal.

Of the 35 data sets, 13 were supplied by coal companies in voluntary support of this study; 8 (U-14 thru U-21) were taken from a published report[1] on the Navajo Generating Station; and 1 (U-22) was extracted from Federal Power Commission reports (FPC Form 423) of utility coal purchases. U-22 deals with coal purchased from coal districts 3, 7, and 8, for which the mean emission level of all purchases by a specific plant was less than 1.2 lb SO_2/MM Btu (the NSPS limit).

A statistical summary of all data sets is given in Section 4, Table 4-1. More detailed statistical summaries are given in Appendix A.

Table 3-1. SUMMARY OF DATA SOURCES

Source of data	Comments
Coal companies	
Appalachian coals	
Data Set C-1	Northern W VA, deep mines, 1972-76. Two-mine operation, crushed to 2 inches, Bradford breaker, shipped to one plant. Raw coal, as loaded, seven continuous and one long wall miners. 702 samples each representing an average of about 12,000 tons of coal.
Data Set C-2	Kentucky, deep mine, 1975. Coal was cleaned at the mine using a jig washer in which the total raw coal stream was cleaned. This is a relatively simple cleaning operation in an older cleaning plant. Sampling of coal was done at the utility plant, 115 samples, each representing about 5600 tons.
Data Set C-3	Kentucky, deep mine, 1975. Coal was cleaned in a modern cleaning plant. The +1/4-inch coal was cleaned in a heavy media vessel, the -1/4-inch coal on a deister table. Filter cake -28M was rejected. 157 samples, each representing about 2500 tons, shipped to one plant.
Other coals	
Data Set C-4	Montana, 1975-76. Two-seam operation with blending of each seam or possible loading of each seam separately. This depends entirely upon the mining

Table 3-1. (continued)

Source of data	Comments
	operation and geological factors. Raw coal shipments, no cleaning. 370 samples, each representing about 10,000 tons, shipped to one plant.
Data Set C-5	Arizona, 1974-1976. Multiple seam operation with all or part of the seams loaded simultaneously. Raw coal shipments with the as-mined coal passing through a Bradford breaker to remove large rock. No further cleaning. 316 samples, each representing about 3000 tons, shipped to one plant.
Data Set C-6	Montana, Core data for one seam of data set C-4, raw coal, 72 cores.
Data Set C-7	Montana, Core data for other seam of data set C-4, raw coal, 110 cores.
Data Set C-8	SE Ohio, Deep mine, 1973-76. One mine operation crushed to 4 inches, no picking, shipped to one plant. ASTM sampling and analysis as loaded. Raw coal, 8 continuous miners, 272 samples, each representing about 10,000 tons.
Data Set C-9	Western PA, Upper Freeport Seam. Core data, raw coal.
Data Set C-10	Central Utah Coal Reserve, I & J Seams. Core data, bottom bench, raw coal.
Data Set C-11	Central Utah Coal Reserve, I & J Seams. Core data, top bench, raw coal.

(continued)

Table 3-1. (continued)

Source of data	Comments
Data Set C-12	SE Ohio, Seams 8 and 9. Raw, strip mining coal, 3 large shovels. ASTM sampling and analysis as loaded. (559 samples, each representing about 10,000 tons.)
Data Set C-13	Wyoming. Core data, raw coal. 140 cores, raw coal.
Utility companies:	
Appalachian coals	
Data Set U-1	As received at one plant, coal as-burned. 103 weekly averages, 721 samples each representing about 5700 tons of coal. 1973-1975.
Data Set U-2	Eastern KY, 1972-1975. Raw coal, as received at one plant. Sampling and analysis at plant, ASTM procedure. 312 samples each representing about 11,000 tons.
Data Set U-3	Eastern Kentucky, 1972-1975. Raw coal, same coal as above, sampling as burned. 103 samples, each representing about 40,000 tons.
Data Set U-4	Eastern Kentucky, 1974-1976. Raw coal, as received at one plant. Modified ASTM sampling at plant. 162 samples each representing about 12,000 tons.
Data Set U-5	Eastern Kentucky, 1974-1976. Raw coal, as received at one plant. Modified ASTM sampling at plant. 250 samples each representing about 13,000 tons.

(continued)

Table 3-1. (continued)

Source of data	Comments
Data Set U-6	West Virginia, District 7, 1975-1976. As received at one plant. 90 samples each representing about 2500 tons.
Data Set U-7	West Virginia, District 7, 1974. As received at one plant. 120 samples, each representing about 1800 tons.
Data Set U-8	West Virginia, District 7, 1974. As received at one plant. 61 samples, each representing about 1000 tons.
Data Set U-9	West Virginia, District 7, 1974. As received at one plant. 74 samples, each representing about 1500 tons.
Data Set U-10	West Virginia, 1975. As received from one supplier for one plant. Hand sampled in part, ASTM for almost all samples. 50 samples each representing about 20 cars on the average.
Data Sets U-11, and U-13	Pennsylvania coal, 1975. Run-of-mine coal, U-12 receives sizing operation in a Bradford breaker which reduces the coal to 1 1/4-inch maximum while removing some shale in the process. Magnetic pulley removes any tramp iron in the raw coal stream. All three data sets - single mine operation. Raw coal. About 250 samples each representing about 4000 to 12000 tons.
Other Coals	
Data Set U-14[a]	Arizona, Core data. 82 core samples, raw coal.
Data Set U-15	Arizona, Core data. 83 core samples, raw coal.

(continued)

Source of data	Comments
Data Set U-16	Arizona, Core data. 395 core samples, raw coal.
Data Set U-17	Arizona, Core data. 132 core samples, raw coal.
Data Set U-18	Arizona, As received data corresponding to U-14. 770 samples.
Data Set U-19	Arizona, As received data corresponding to U-15. 113 samples.
Data Set U-20	Arizona, As received data corresponding to U-16. 768 samples.
Data Set U-21	Arizona, As received data. 1766 samples.
Data Set U-22	Districts 3, 7, and 8 purchases by utilities during 1974 and 1975. FPC form 423 data. 63 samples, each representing all purchases for which the mean lb SO_2/MM Btu for one plant was less than 1.2.

[a] Data sets U-14 through U-21 are described in greater detail in reference 1.

4.0 DATA ANALYSIS

The methods of data analysis are described in Appendix B, except for some standard methods that are given in statistical texts.[5,6,7] The data sets are presented in Appendix A, with results of pertinent analyses. This section summarizes the results of the analyses.

Table 4-1 summarizes sulfur variability data by data set. The arithmetic mean (or average) and the standard deviation of the measurements of sulfur content are given in columns 2 and 3. The relative standard deviation* (RSD) is the ratio of the standard deviation to the mean, expressed as a percentage, and is given in column 4. For example, for Data Set C-5, the mean is 0.42, the standard deviation is 0.048, and the RSD is 11.5 percent. There is a preference for using the RSD's when the standard deviation depends on the average concentration. Comparison of the RSD's for the core data with those of the run-of-mine data indicates clearly that the core data show larger variations by factors ranging from 1½ to 4, as illustrated in Table 4-2. This is an expected result; the core sample represents a very small amount of coal relative to a composite sample from a unit train of several thousand tons.

Similar analyses were performed with respect to heating value or Btu content. It is readily apparent that the RSD of Btu content is much smaller, 2 to 5 percent, than that for sulfur content which is 5 to 25 percent.

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*The RSD is also referred to as the coefficient of variation.

Data set	Weight percent sulfur			Btu/lb			lb SO_2/MM Btu			Core data
	Average	Standard deviation	RSD[a] of variation, %	Average	Standard deviation	RSD of variation, %	Average	Standard deviation	RSD of variation, %	
C-1	2.30	0.224	8.0	13,050	224	1.7	4.21	0.34	8.1	
C-2	0.60	0.030	5.0	12,310	161	1.3	0.93	0.07	7.1	
C-3	0.81	0.088	10.9	12,250	276	2.3	1.26[b]	0.14	11.1	
C-4	1.38	0.253	18.3	11,540	263	2.3	2.34	0.43	18.4	
C-5	0.42	0.049	11.7	12,150	197	1.6	0.65[b]	0.08	11.8	
C-6	2.14	0.705	33.0	11,540	291	2.5	3.52[b]	1.20	33.1	yes
C-7	1.50	0.405	27.0	11,480	289	2.5	2.48[b]	0.67	27.1	yes
C-8	2.60	0.130	5.0	12,480	708	5.7	4.08	0.30	7.6	
C-9	1.63	0.780	47.8	14,190	142	1.0	2.25	1.08	47.8	yes
C-10	0.58	0.247	42.6	13,170	332	2.5	0.88	0.38	42.7	yes
C-11	0.88	0.535	60.7	12,700	291	2.3	1.39	0.86	60.7	yes
C-12	3.42	0.344	10.1	12,000	160	1.3	5.69	0.59	10.4	
C-13	0.58	0.090	15.5	11,970	191	1.6	0.95	0.15	15.6	yes
U-1	0.74	0.129	17.4	12,500	250	2.0	1.16	0.21	17.5	
U-2	0.88	0.103	11.7	12,000	250	2.1	1.39[b]	0.17	11.9	
U-3	0.82	0.160	19.5	11,000	210	1.9	1.42[b]	0.28	19.6	
U-4	1.04	0.180	17.3	12,000	280	2.3	1.73	0.29	16.2	
U-5	0.93	0.170	18.3	11,900	280	2.4	1.55	0.18	11.6	
U-6	0.79	0.150	19.0	11,800	400	3.4	1.34	0.27	20.1	
U-7	0.80	0.170	21.3	12,300	488	4.0	1.29	0.28	21.7	
U-8	0.84	0.220	26.2	12,200	636	5.2	1.37	0.34	26.7	
U-9	0.81	0.160	19.8	12,000	665	5.5	1.35	0.27	20.0	
U-10	1.74	0.59	33.9	12,070	382	3.2	2.83	0.96	34.1	
U-11	3.10	0.366	11.8	11,030	332	3.0	5.34[b]	0.65	12.2	
U-12	2.25	0.157	7.0	11,770	251	2.1	3.36[b]	0.25	7.4	
U-13	2.15	0.152	7.1	11,480	267	2.3	3.56[b]	0.27	7.6	
U-14	0.49	0.195	39.8							yes
U-15	0.53	0.164	30.9							yes
U-16	0.68	0.180	26.5							yes
U-17	0.66	0.180	27.3							yes
U-18	0.37	0.035	9.5							
U-19	0.46	0.055	12.0							
U-20	0.60	0.092	15.3							
U-21	0.55	0.086	15.6							
U-22							1.03[c]	0.16	15.8	

[a] RSD (relative standard deviation) is the ratio of the standard deviation to the average, multiplied by 100 to yield the standard deviation as a percent of the average.

[b] Estimated from weight percent sulfur and Btu/lb data, not by direct computation.

[c] Positive square root of pooled variance estimates, coals purchased in Districts 3, 7, and 8, for which the mean or average sulfur content is less than 1.2 lb SO_2/MM Btu.

Analysis of sulfur content in lb SO_2/MM Btu was performed by two methods:

1. Using lb SO_2 computed by multiplying weight percent by 1.90 (lb SO_2 emitted per lb sulfur in coal), dividing by the Btu content, and expressing the results in lb SO_2/MM Btu.
2. By direct use of results from analysis of weight percent sulfur and Btu/lb.

In case 2, the RSD of lb SO_2/MM Btu is given by the approximation, square root of the sum of squares of the RDS's of weight percent sulfur and Btu content. See Appendix B.1.5 for the explanation of this method. Also in case 2, the average lb SO_2/MM Btu is given by the quotient of the corresponding average of weight percent sulfur $\times$ 1.90 by average Btu/lb $\times 10^{-6}$.

Table 4-2. COMPARISON OF AVERAGE AND RSD OF SULFUR CONTENT OF RUN-OF-MINE COAL AND CORRESPONDING CORE DRILLING SAMPLES

Run-of-mine data			Core drilling data		
Data set	$\bar{x}$	RSD, %	Data set	$\bar{x}$	RSD, %
C-4	1.38	18.3	C-6	2.14	33
			C-7	1.50	27
U-18	0.37	9.5	U-14	0.49	39.8
U-19	0.46	12.0	U-15	0.53	30.9
U-20	0.60	15.3	U-16	0.68	26.5

Standard frequency tabulations were made of almost all the data sets with low sulfur content, less than or equal to 1 percent. The tabulation for Data Set U-1 is given in Table 4-3. A histogram and a cumulative frequency distribution are given for these same data in Figures 4-1 and 4-2, respectively. The cumulative frequency is plotted on logarithmic normal probability paper because the data

Table 4-3. FREQUENCY TABULATION OF WEEKLY WEIGHTED AVERAGE OF SULFUR (DRY BASIS) FOR DATA SET U-1[a]

Weighted average, %	Frequency
0.62 - 0.63	1
0.64 - 0.65	6
0.66 - 0.67	5
0.68 - 0.69	14
0.70 - 0.71	21
0.72 - 0.73	13
0.74 - 0.75	6
0.76 - 0.77	11
0.78 - 0.79	8
0.80 - 0.81	3
0.82 - 0.83	2
0.84 - 0.85	4
0.86 - 0.87	1
0.88 - 0.89	4
0.90 - 0.91	0
0.92 - 0.93	1
0.94 - 0.95	1
0.96 - 0.97	1
0.98 - 0.99	1
	$\bar{x} = 0.742$
	$s = 0.0732$

[a] Each weekly weighted average is a weighted average of the seven daily measurements of weight percent sulfur for a composite sample from an average of about 5700 tons of coal.

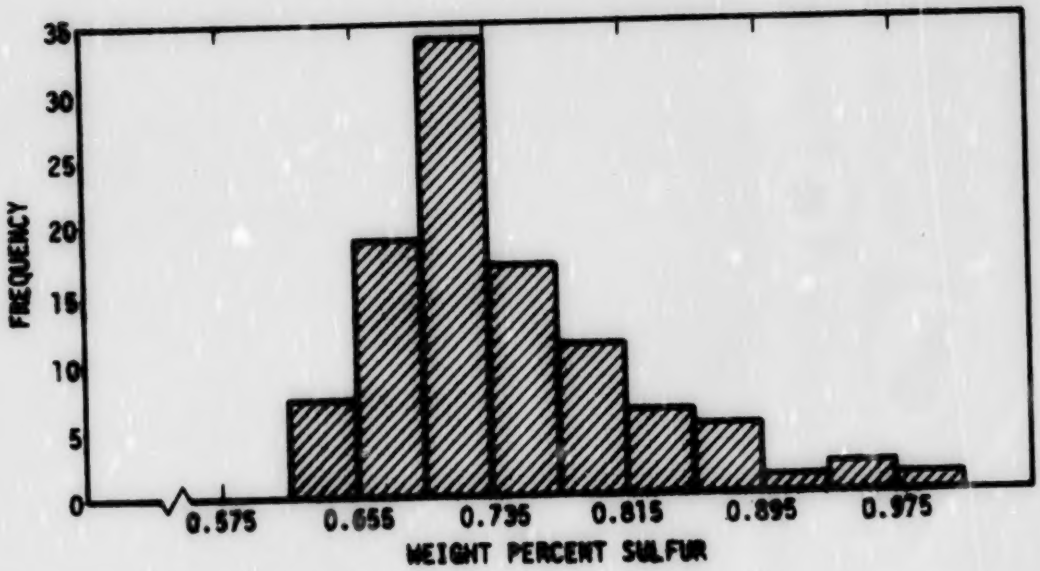

Figure 4-1. Histogram of weekly averages for data set U-1.

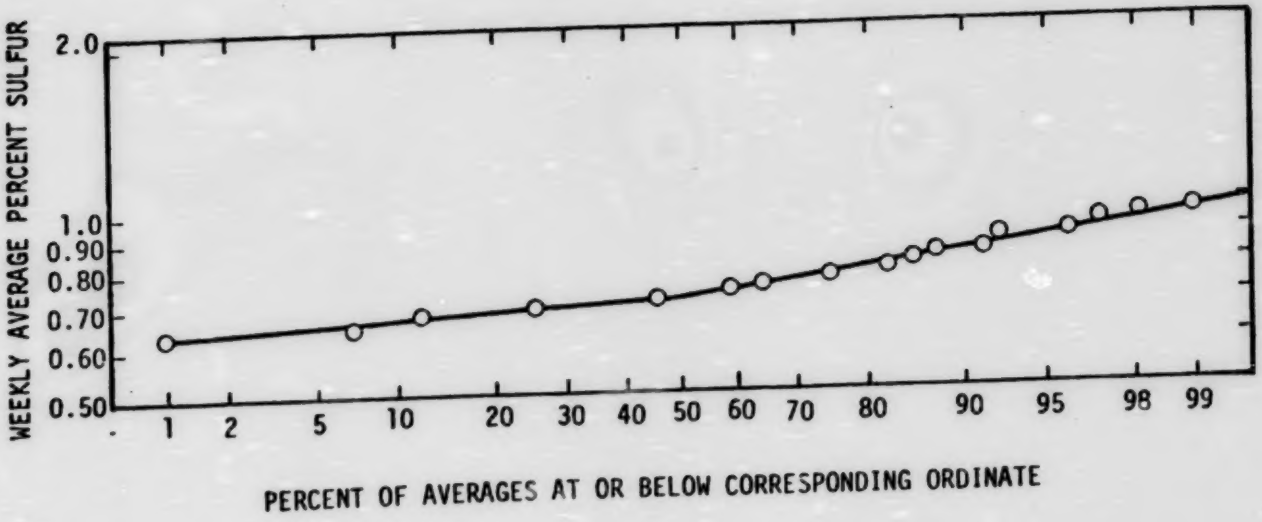

Figure 4-2. Cumulative frequency graph of data set U-1

are skewed toward the high values. This skewing is obvious upon examination of the histogram in Figure 4-1, which shows tail-off of the data at the higher values of weight percent sulfur. Reference 1 discusses this point in considerable detail and suggests the inverted gamma frequency distribution. The lognormal is another good empirical approximation, which is more convenient than the inverted gamma and gives results that are identical for all practical purposes. Rational explanations for the logarithmic normal distribution are given in statistical literature.[6]

One other graphical analysis that is useful in interpreting the variability of sulfur content in coal is a time series plot of the sulfur measurements by unit train and by weekly average, as shown in Figure 4-3 for Data Set U-1. This plot allows comparison of short-term variation (week-to-week) and a 4-week moving average. The definite tendency is for the values of successive weeks to be closer than those that are more separated in time (that is, a greater correlation among the weight percent sulfur values with a short time lag). No sophisticated time series analyses were conducted because the time increments between unit train samples are not typically the same within data sets. Data set U-1 is unique in this respect in that there is one data value for each day.

Several analyses were made of the correlation between the sulfur and Btu contents of the coal; these were done by a combination of cross-tabulation and computation of the correlation coefficient. Results are given in Appendix A (cross-tabulations) and Appendix B (for Data Set U-1).

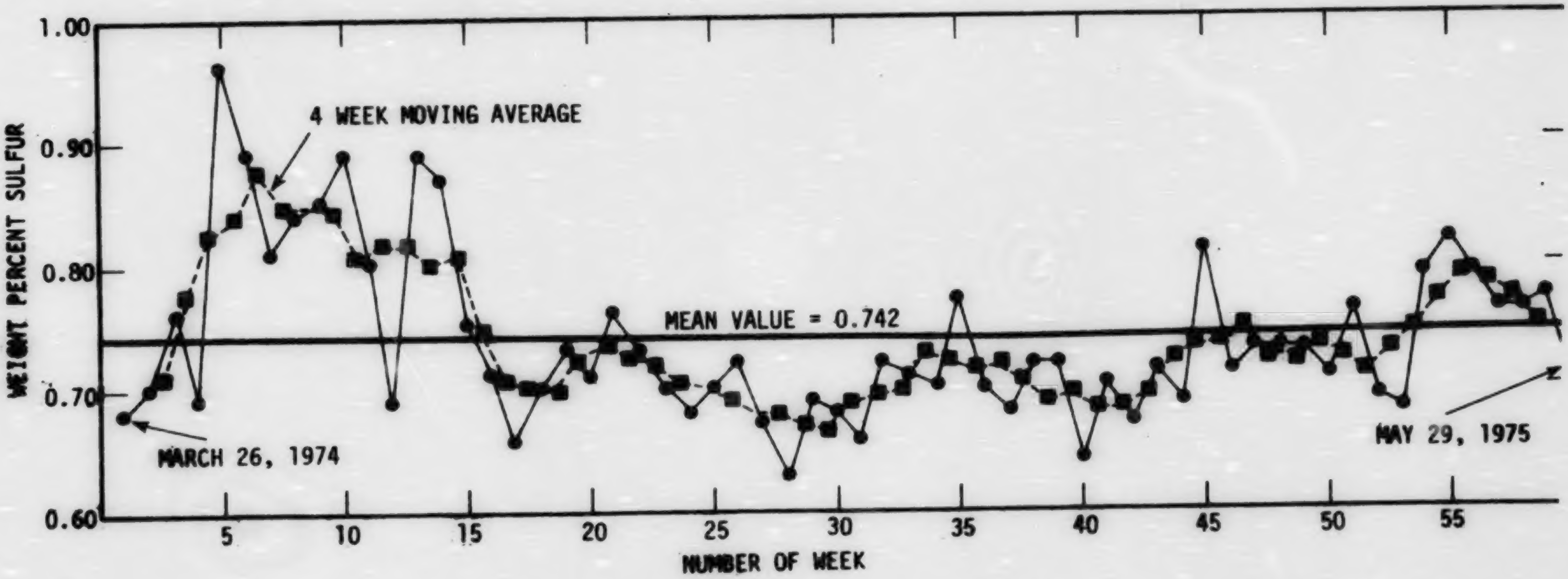

Figure 4-3. Average weekly sulfur content of coal of data set U-1 vs. the number of the week.

5.0 IMPLICATIONS OF STATISTICAL ANALYSIS

5.1 BACKGROUND AND INTRODUCTION

The purpose of this section is to provide a methodology for estimating the impact of the variability of sulfur as a function of the tons (e.g., per 3 hours) and/or the number of composite samples analyzed and averaged (per week, month, etc.) on determination of compliance with SIP or NSPS emission regulations. Since the impact will be similar for both types of regulations, only NSPS compliance is considered here. (An example is given for a regulation other than NSPS.)

THE NSPS 1.2 lb SO_2/MM Btu, is stated as a maximum value never to be exceeded. A legal interpretation of the NSPS would require that at no instant of time would the value 1.2 be exceeded. An interpretation of the NSPS based on a continuous monitoring procedure is that the maximum 3-hour average should not exceed 1.2 lb SO_2/MM Btu. A typical plant of 500 MW capacity will burn about 600 tons of coal in three hours. The smallest plant covered by the NSPS, 25 MW, will burn about 33 tons of coal in 3 hours. Although no data sets in this study contain samples that represent an amount of coal as small as either 600 or 33 tons, data set C-12 and other selected data sets provide statistical evidence of a trend toward smaller RSD's as the quantity (tons) of coal sampled increases. (See Appendix C for a detailed discussion of this relationship.) This tendency is consistent with that expected from the standpoint of a physical explanation of the sampling process (see ASTM procedure[3]). The available data are extrapolated to yield a reasonable estimate of the RSD as a function of quantity of coal sampled in order to estimate the impact of the variability in sulfur content on compliance with NSPS. It will be more difficult for a small plant to comply with the NSPS, on the assumption that the coal burned during each 3-hour period will vary more in sulfur content than will that burned at a typical 500-MW plant. This problem is discussed further in Section 5.2.

If the interpretation of NSPS involved a longer period, such as 1 week or 1 month, and if several composite samples were analyzed and the results averaged for the longer period of compliance, then the impacts on compliance with NSPS can be estimated by use of the data and methods presented in this section.

Compliance in this case would be determined by averaging, for example, all the composite analyses of sulfur content for the specified period, say 1 month. If this average is less than some specified value, such as 1.2 lb SO_2/MM Btu, one would infer that the coal would provide compliance. In this case, the variability of the average depends on (1) the amount of coal represented by each composite sample and (2) the number of composite samples averaged during the specified period. The results will vary less as either the tonnage and/or the number of samples increases. The smaller plants are expected to receive smaller quantities per sample as well as fewer samples per month, and hence, the averages for a specified period of 1 month will vary more than those of a larger plant. A methodology is presented wherby an analyst or company may use estimates of the RSD based on computations with available data or perhaps by computer simulation models to yield a measure of the likelihood of compliance with NSPS or other emission regulations.

As an illustration of the determination of the impact of sulfur variability on compliance with the NSPS, consider the following example. A typical 500-MW plant uses 600 tons of coal per 3 hours, 4800 tons per day, and receives coal four times per week (four unit trains), averaging 8400 tons each delivery. Thus 16 composite samples are analyzed for lb SO_2/MM Btu averaged for 1 month. If this monthly average is used as a basis for compliance, what is the expected frequency with which the average exceeds 1.2 lb SO_2/MM Btu? What is the average required sulfur content (weight percent) that will enable compliance with the regulation assuming that variability is (1) as estimated in this study and (2) as estimated from data available to the company?

Although NSPS does not legally permit a single value to exceed a standard, in practice it is necessary to admit noncompliance for some minimal number or percentage of cases (averaging periods). The percentage of noncompliance can be maintained at a small value that will provide a reasonable trade-off between availability of coals (costs) and average emission levels. Several hypothetical examples are posed at the end of this section to illustrate use of the methodology.

5.2 DATA ANALYSIS

Analyses were performed on the data to (1) relate the variability of sulfur content to the amount (tonnage) of the coal from which the composite sample was taken and/or the number of composite samples used in obtaining the average, (2) estimate the form of the frequency distribution of the individual analyses of composite samples from unit trains, and (3) estimate the impact of sulfur-content variability on the determination of compliance with emission regulations.

Appendix B.1.2 presents a brief description of the statistical methods and a summary of the means and the estimates of the standard deviations of the several sources of variation corresponding to the number of composite samples analyzed and averaged during one week, month, etc. All of these analyses were performed using the data in their original scale, that is, with no transformations. Logarithmic transformations are often used to stabilize the variance if the standard deviation in the original scale varies directly as the mean (see Figure B-2). In several analyses essentially the same result was obtained with and without the transformation. It can be readily established that the standard deviation of ln x is approximately equal to the RSD of x. Thus the use of RSD's results in a stabilization of the variance in situations where the logarithmic transformations are indicated. The RSD's are also more useful for making comparisons in the study of the variability of sulfur in coal. At the bottom of Table B-5 are given the weighted average or pooled standard deviations and the average relative standard deviations. These are typical values that might occur for a particular application. The coals represented in this table have had different degrees of preparation and cleaning and have been mined by different techniques. If one knows that the mining and processing techniques of a given operation will provide a coal of less (or greater) variation in sulfur content than the average, then the lower (or upper) limits in Table B-6 might be used, as described in Appendix B.1.3. NOTE: It is emphasized that the values are based on a collection of coal data made available by selected companies. Each company using the approach presented herein is urged to use its own data in estimating the variability of sulfur content for specified averaging times or tonnages.

As stated earlier, the data for estimating sulfur variability are based on analyses of composite samples from unit trains in quantities ranging from about 1,000 tons to 20,000 tons; none of the data extended to the range of 33 tons or 600 tons, as required to estimate the impact of variability on compliance in a 3-hour averaging time for either a small plant or an average plant. Thus, an attempt is made to extrapolate the available data to smaller amounts of coal (in tons) by use of specific data sets as a basis for obtaining a relationship between the relative standard deviation and the amount of coal sampled. For the region of extrapolation, this relationship is assumed to be linear in the logarithm of the amount of coal sampled in tons, i.e., the RSD is given by

$$RSD = a + b \log_{10} T \qquad \text{(Equation 5-1)}$$

where

RSD = relative standard deviation of weight percent sulfur,

T = tons of coal sampled,

$\log_{10} T$ = log of T, base 10 (common logarithms), and

a,b = constants to be determined by the data.

Data set C-12, gives the following results for the relative standard deviation as a function of the amount of coal sampled.

Weight percent sulfur					
Amount of coal sampled, tons	Minimum	Maximum	Mean, $\bar{X}$	Standard deviation, s	RSD = $s/\bar{X}$
2,000	1.76	5.04	3.42	0.4251	0.1243
10,000	2.54	4.34	3.42	0.3436	0.1005

Using these data and equation 5-1, one obtains

$$0.1005 = a + b \log_{10} 10{,}000$$

$$0.1243 = a + b \log_{10} 2{,}000,$$

and hence, upon solving for a and b,

$$a = 0.237$$

$$b - -0.0341,$$

and

$$RSD \approx 0.289 - 0.0341 \log_{10} T.$$

For this particular coal (Data Set C-12) the RSD is slightly smaller than the average for all of the coal data analyzed; hence, the value of RSD was adjusted by increasing the constant term, a, from 0.237 to 0.289. Thus the final equation for RSD_p, i.e., the predicted RSD, is

$$RSD_p = 0.289 - 0.0341 \log_{10} T. \quad \text{(Equation 5-2)}$$

Because this equation is based on a small amount of data, it is desirable to extrapolate the result to the extreme, comparable to a core sample representing, say, 50 pounds of coal. For T = 50/2000 = 0.0250,

$$RSD_p = 0.289 - 0.0341(\log 0.025) = 0.344,$$

which is consistent with the core sample data. That is, in Table 4-1 the RSD's for core sample data (indicated by a yes in the last column) range from 26 to 48 percent and one value is at 61 percent. Thus, the 34 percent value is a reasonable extrapolation of the results.

These results are given in Table 5-1 above the dashed line for a 25-MW plant (33 tons of coal burned in 3 hours) and below the dashed line for a 500-MW plant (600 tons of coal burned in 3 hours, 4800 tons per day, 4 unit trains per week, 16 sample results averaged per month). Figure 5-1 also shows the impacts on the RSD of the weight percent sulfur of the amount of coal represented by a composite sample and/or the number of sample results averaged during the specified averaging period. The extrapolation to 33 tons for the 25-MW plant is shown by a dashed line. At the extreme right-hand portion of the curve it is desired that the RSD tend to zero as the averaging time approaches the life of the typical mine, say 20 years. The RSD for the left-hand portion of the curve approaches that of core samples. Hence, the two ends of the curve in Figure 5-1 are reasonably well determined, as are some of the central portions, particularly for the amount of coal represented by one unit train and for averages of results of 16 composite samples per month.

5.3 AVERAGE SULFUR CONTENT REQUIRED FOR COMPLIANCE

Computation of the average sulfur content required for compliance with the 1.2 lb SO_2/MM Btu regulation is based on appropriate assumptions concerning the frequency distribution of the data. The computations assume a typical 500-MW plant or 25-MW plant, as indicated.

5.3.1 Averaging Period of One Month, Assuming Normality, 500-MW Plant

Let m be the mean required sulfur content, then for 16 samples per month, RSD = 0.069 for weight percent sulfur, Table 5-1. It is assumed that the RSD for lb SO_2/MM Btu = 1.05 × RSD for weight percent sulfur. See Appendix B.1.5

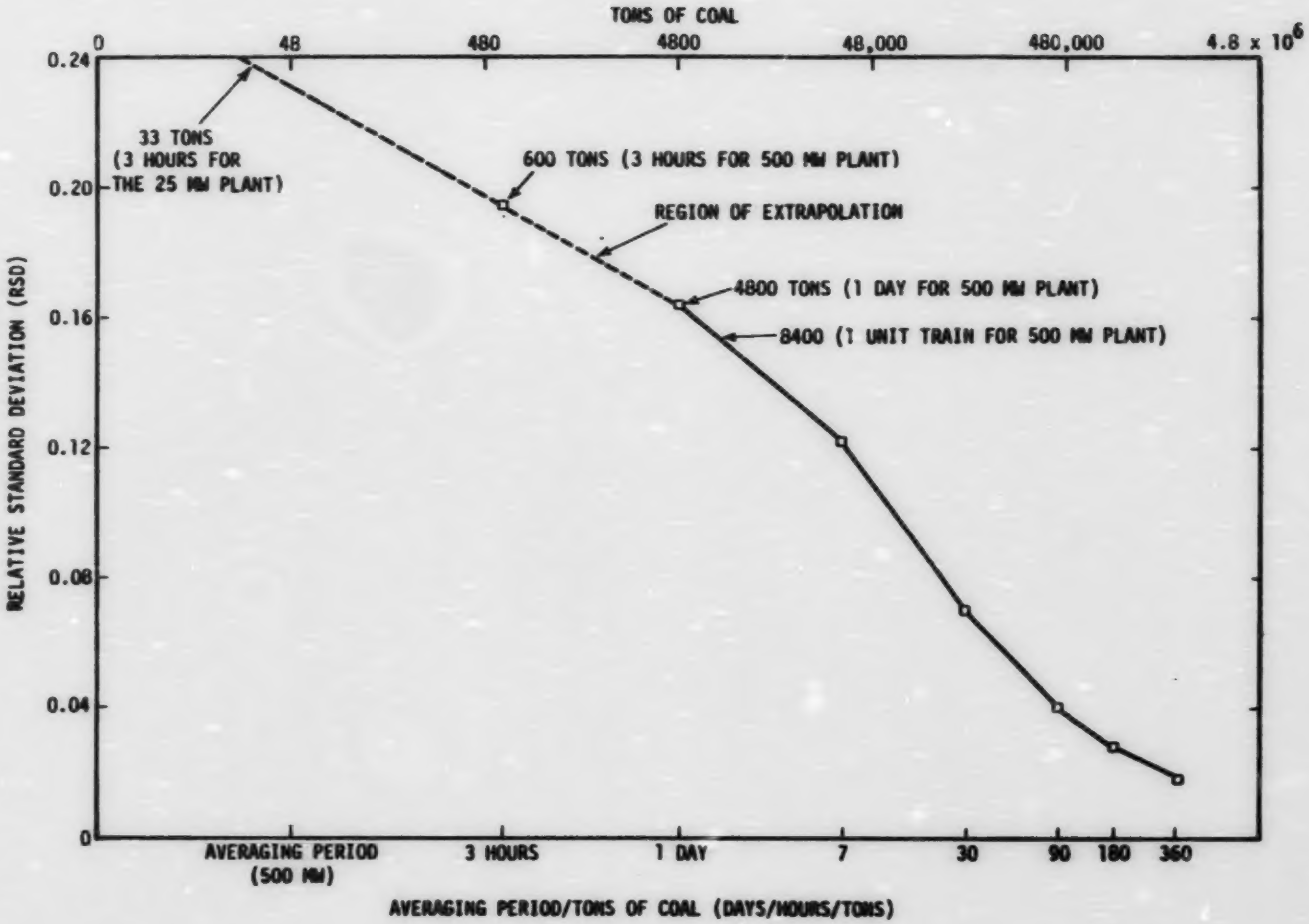

Figure 5-1. RSD versus averaging period/tons of coal (days/hours/tons).

Table 5-1. EXPECTED VALUES OF THE RELATIVE STANDARD DEVIATION[a] OF WEIGHT PERCENT SULFUR VS. NUMBER OF COMPOSITE SAMPLES PER INDICATED AVERAGING PERIOD/TONS

(Based on data for coals with 1.0 percent sulfur or less)

Averaging period/(tons)	Expected variation (RSD)				
3 hours / 33 tons, 25 MW plant	23.7%				
3 hours / 600 tons, 500 MW plant	19.4%				
Among unit trains (composite samples of 8400 tons, 500 MW plant)	15.5%				
		w = 1[b]	w = 2	w = 4	w = 7
Weekly averages		15.5%	14.2%	12.3%	11.4%
	n = 2[c]	n = 4	n = 8	n = 16	n = 28
Monthly averages	11.7%	9.3%	7.8%	6.9%	6.5%
Quarterly averages	6.8%	5.4%	4.5%	4.0%	3.8%
Semiannual averages	4.8%	3.8%	3.2%	2.8%	2.7%
Annual averages	3.4%	2.7%	2.3%	2.0%	1.9%

[a] The relative standard deviation can be used for a wide range of levels of sulfur content. For example, if the coal is 1 weight percent sulfur, the values in the table also serve as the absolute standard deviation; if, however, the coal is 1.5 percent sulfur, the values should be multiplied by 1.5 to yield the absolute standard deviation. The data in this study cover the range up to 1.5 percent, and these percentages seem to be appropriate over this range. See Figure B-2.

[b] w is the number of unit trains (composite samples averaged) per week.

[c] n is the number of unit trains (composite samples averaged) per month.

for a discussion of this relationship between the RSD's. Hence, the RSD for lb SO_2/MM Btu for averages of composite samples from 16 unit trains, is given by

$$0.069(1.05) = 0.0725.$$

Figure 5-2 illustrates the determination of the value m, in that for 95 percent compliance the distance between m and 1.2 must be equal to 1.645 σ where σ is the absolute standard deviation of lb SO_2/MM Btu and is 0.0725 m, i.e., the product of the RSD by the mean value, m. The value 1.645 is obtained from a table for areas under a standard normal curve, corresponding to the 95th percentile.[8]

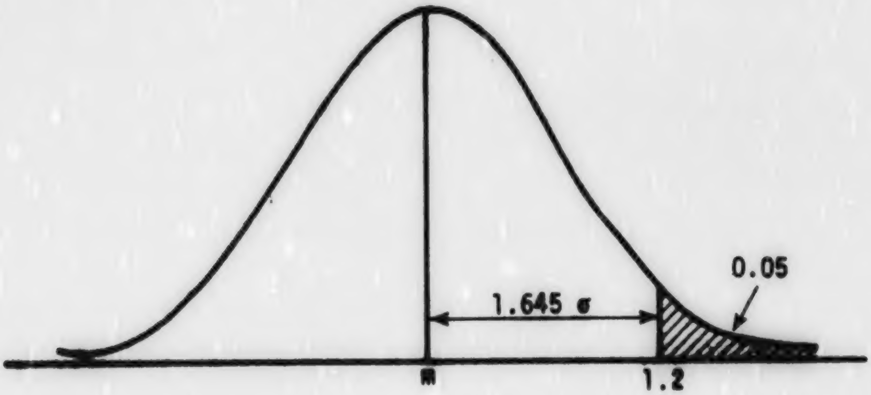

Figure 5-2. Determination of required average sulfur content

Hence,

$$u = \frac{1.2 - m}{0.0725m} = 1.645 \qquad \text{(Equation 5-3)}$$

or

$$m = 1.072.$$

The weight percent sulfur for a coal with heating value of 11,500 Btu/lb is given by 0.727 × (1.072/1.2) = 0.65. The value is then adjusted by the ratio 1.072/1.2. Similarly the calculation of m for 99 percent compliance is made by using 2.326 in place of 1.645. Values of m for different amounts of coal sampled/numbers of samples averaged are obtained by multiplying the RSD given in Table 5-1 by 1.05 and substituting it for the 0.0725 in equation 5-3.

5.3.2 Averaging Period of One Month, Assuming Lognormal Distribution, Small Plant

In this case, the number of composite samples averaged per month will be smaller than that for the 500-MW plant. For example, if 33 tons are required per 3 hours, 264 tons per day, or 7920 tons per month, it is possible that the entire month's supply would be obtained at one time. For the purpose of illustration, assume that four shipments of about 2000 tons each are received in one month and that the average sulfur content of four composite samples is used to determine compliance with the emission regulation. From Figure 5-1, the expected RSD of weight percent sulfur for samples of 2000 tons is about 18.2 percent. On the assumption that four composite samples are averaged each month, from Table 5-1 we read that RSD is 9.3 percent. However, this is based on samples from a larger tonnage of coal, 8400 tons rather than the 2000 in this example. Hence, assuming the same percentage reduction in the RSD is given for 4 composite samples from the 2000 ton lot as for the 8400 ton lots, an estimate of the RSD is the product of 18.2 by (9.3/15.5) or about 10.9 percent. The RSD of lb SO_2/MM Btu is estimated to be 11.5 percent (= 10.9 × 1.05). This value is then used along with the lognormal distribution to obtain an estimate of the required mean sulfur content.

The normality assumption is not good for averages of a small number of measurements, say less than 5; hence, the lognormal distribution was assumed for this example.

The lognormal distribution is skewed toward the larger values as shown in Figure 5-4, upper sketch. The lower distribution is that of the transformed variable, $Y = \ln X$, which should be approximately normally distributed. As stated earlier, the lognormal distribution provides a very good empirical fit to several of the data sets, and because of its mathematical convenience, the impact of sulfur content is first determined by use of this distribution. The value of 0.115, is determined as follows: If $Y = \ln X$, and the standard deviation of X is (1.05) × 10.9% or 0.115, where m is the mean of X, m' the mean of Y, then the standard deviation of Y is approximately,

$$\sigma\{Y\} \approx \frac{1}{m}\,\sigma\{X\} = \mathrm{RSD}\{X\}.$$

That is, the estimated standard deviation of lnX is essentially equal to the RSD of X. Hence, the transformation $Y = \ln X$ is a variance stabilizing one, provided the RSD's are approximately the same for all levels of sulfur content. Calculations using the RSD of the sulfur content are essentially equivalent to those using the standard deviation of the logarithm of sulfur content.

From Figure 5-3, the sulfur content required for 95 percent compliance with 11,500 Btu/lb heating value would be 0.727 × (0.993/1.2) = 0.60 weight percent sulfur. The multiplication by 0.993/1.2 provides for the adjustment from 1.2 to 0.993 as the required average.

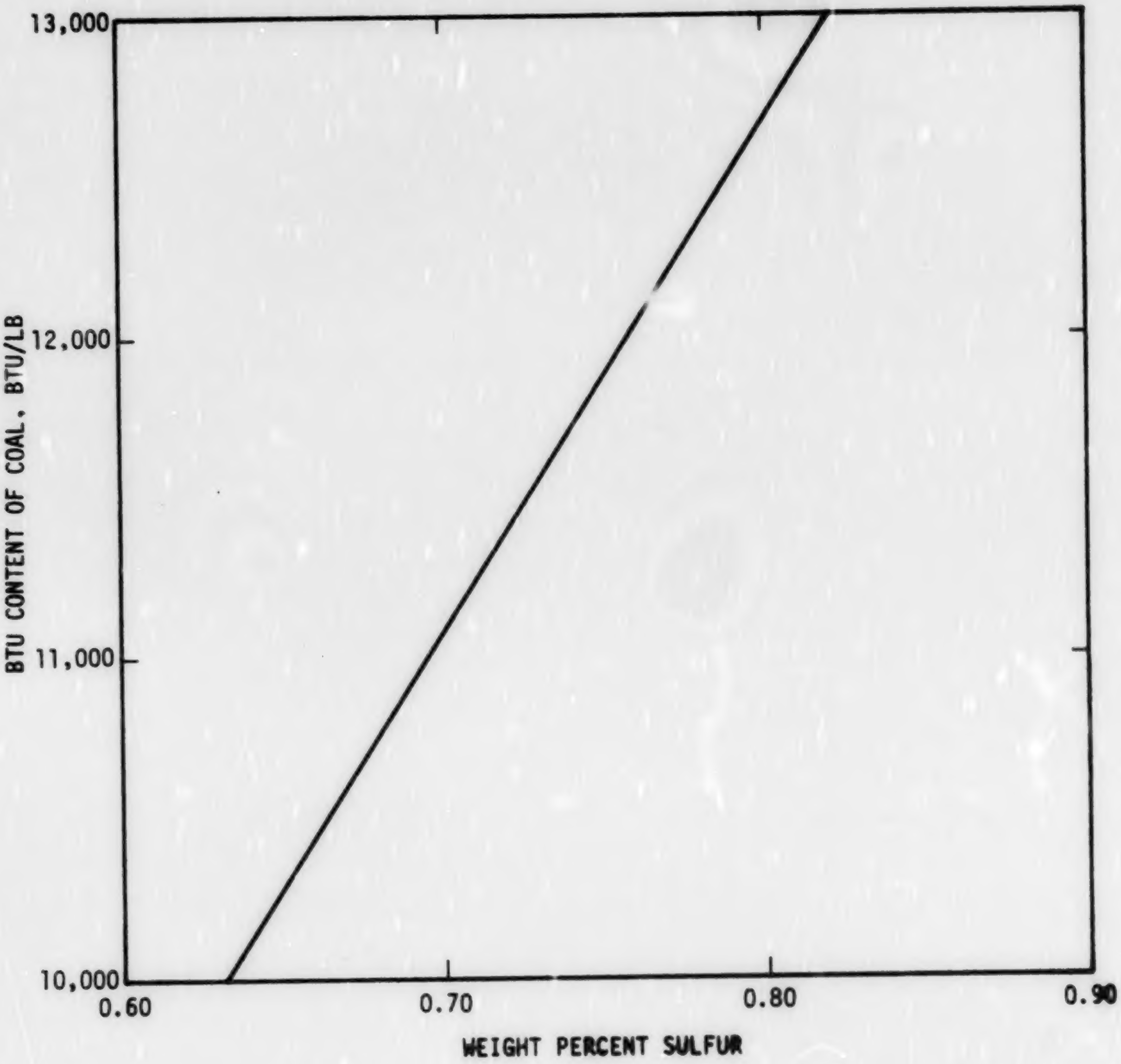

Figure 5-3. Sulfur content versus heating value of coal required to yield 1.2 lb SO_2/MM Btu (1.90 lb SO_2/lb S assumed)

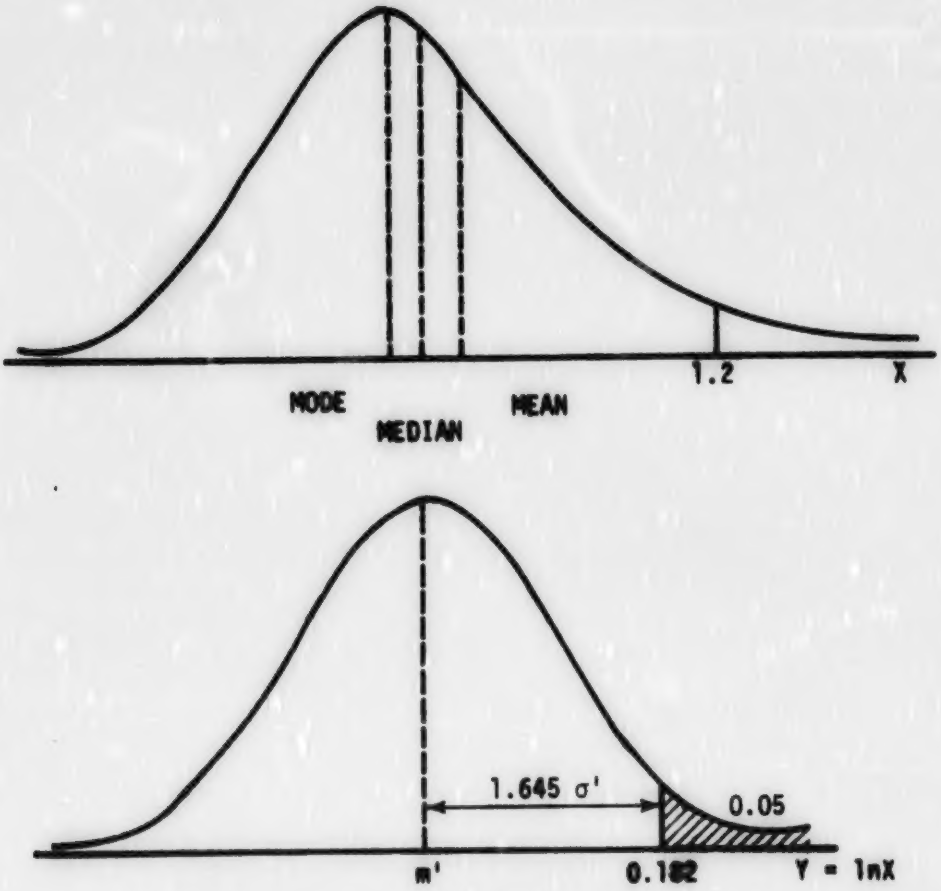
MODE
MEDIAN
MEAN
1.2
X
1.645 σ'
0.05
m'
0.182
Y = lnX

Figure 5-4. Determination of required average sulfur content assuming lognormal distribution

Referring to Figure 5-4, lower sketch, the following approximate relationship applies, where m' is the mean of the lognormal distribution, 0.182 = ln 1.2.

$$u = \frac{0.182 - m'}{0.115} = 1.645$$

$$m' = -0.0072$$

$$m \approx e^{m'} = 0.993 \text{ lb } SO_2/\text{MM Btu.}$$

5.3.3 Three-Hour Average, Lognormal Distribution, 500-MW Plant

Now consider the implication of the expected variation in 3-hour averages for the 500-MW plant with respect to compliance with the NSPS. In this case the graph of Figure 5-1 provides an estimate of the RSD of the weight percent sulfur to equal to 19.4 percent, or 19.4 × 1.05 or 20.4 percent for lb SO_2/MM Btu. Using the same procedure as in earlier computations

$$u = \frac{0.182 - m'}{0.204} = 1.645$$

$$m' = -0.1536$$

$$m \simeq e^{m'} = 0.858 \text{ lb } SO_2\text{/MM Btu.}$$

Using Figure 5-3, the weight percent sulfur assuming 11,500 Btu/lb, must average 0.727 (0.858/1.2) = 0.52 weight percent sulfur.

This computation is also performed in Appendix B.3 assuming the inverted gamma distribution; the result is 0.875 compared with the 0.858 obtained using the lognormal distribution. This same close relationship was found for a large number of cases computed with both lognormal and inverted gama distributions.

5.3.4 Three Hour Average, Lognormal Distribution, Small Plant

For this case, the RSD as read from Figure 5-1 for 33 tons is about 23.7 percent, or about 24.9 percent for lb SO_2/MM Btu. Hence, the estimated mean for 95 percent compliance is estimated to be about 0.80. In terms of sulfur content, this transforms to 0.727 × (0.80/1.2) = 0.49 average weight percent sulfur.

The results of these four analyses are tabulated in Table 5-2 for convenience of subsequent discussion.

Table 5-2 shows clearly that the mean weight percent sulfur required for compliance with the NSPS decreases as the averaging period changes from 1 month to 3 hours and also as the plant size decreases. As the percentage of compliance increases, the mean will decrease still further and will rapidly approach a value for which the availability of raw coal for compliance with NSPS tends to be very limited.

Figure 5-5 provides a general procedure for determining the average sulfur content of a coal (lb SO_2/MM Btu) required to achieve compliance with the NSPS of 1.2 lb SO_2/MM Btu for 90, 95, and 99 percent of the averages of individual unit train samples for specified RSD's. Results are given for assumptions of both normal and lognormal distribution of sulfur content (lb SO_2/MM Btu). For example, if a company has determined that the monthly averages vary by approximately 7 percent of the mean (RSD = 7%), then for 95 percent compliance, the required mean sulfur content would be about 1.08 (1.07) lb SO_2/MM Btu assuming normal (lognormal) distribution. If one does not have information from which to derive the sulfur variability in terms of the RSD, then Figure 5-1 may be used to obtain an estimated value. These values would be multiplied by 1.05 to estimate the RSD of lb SO_2/MM Btu in terms of that for weight percent sulfur. The RSD's read from Figure 5-1 would be applicable to coals with sulfur content up to 3 percent, the limit of data used in this study. It is expected, however, that the RSD's also would apply to higher sulfur levels.

Table 5-2. SUMMARY OF COMPLIANCE COMPUTATIONS (95 AND 99% COMPLIANCE WITH EMISSION RATE OF 1.2 lb SO_2/MM Btu)

Averaging period	Information	500-MW plant	25-MW plant
1 month	Number of composite samples per month	16	4
	Tonnage of coal represented by each sample	8,400	2,000
	RSD (lb SO_2/MM Btu)	7.25%	11.5%
	Assumed heating value	11,500	11,500
	Required mean (lb SO_2/MM Btu) for 95% compliance	1.072	0.993
	Required mean (weight percent sulfur) for 95% com.	0.65	0.60
	Required mean (lb SO_2/MM Btu) for 99% compliance	1.014	0.918
	Required mean (weight percent sulfur) for 99% com.	0.61	0.56
3 hours	Tonnage of coal represented by each sample	600	33
	RSD (lb SO_2/MM Btu)	20.4	24.9
	Assumed heating value	11,500	11,500
	Required mean (lb SO_2/MM Btu) for 95% compliance	0.86	0.80
	Required mean (weight percent sulfur) for 95% com.	0.52	0.49
	Required mean (lb SO_2/MM Btu) for 99% compliance	0.75	0.67
	Required mean (weight percent sulfur) for 99% com.	0.45	0.41

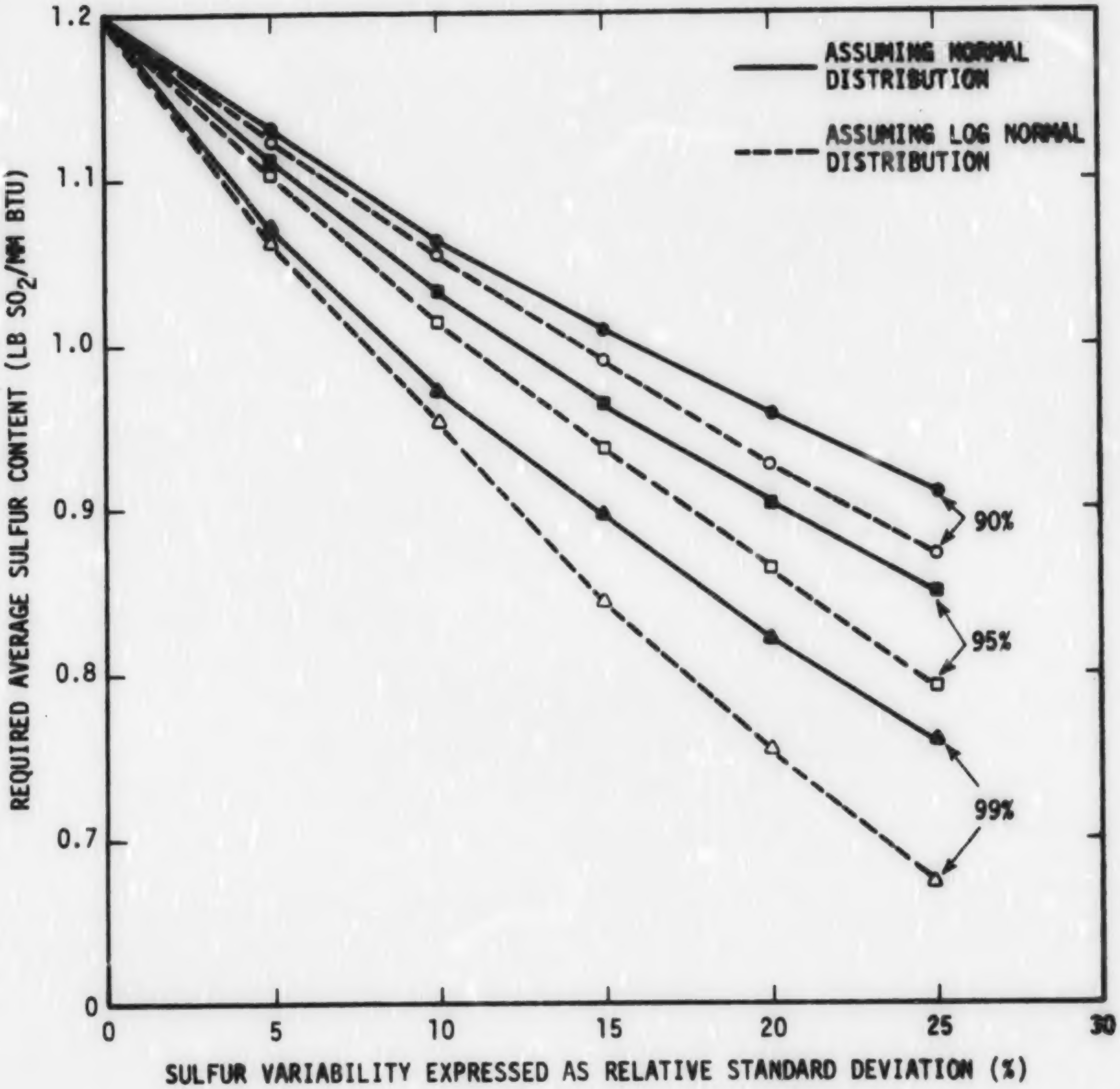

Figure 5-5. Required average sulfur content vs. sulfur variability.

5.4 EXAMPLES

The essence of this report is application of the tables and graphs of this section, together with necessary background information as described in other sections, to real problems of compliance with NSPS or other emission limitations. As an aid in understanding the application of these graphs and tables, several examples are given to represent some typical problems.

Example 1

A coal is assumed to vary in a manner consistent with the data in Figure 5-1, having an average weight percent sulfur equal to 0.70. Assume that four unit trains are received each week, i.e., Table 5-1 under w=4. What is the expected range of variation of sulfur content (weight percent) of 95 percent of the weekly averages (2σ limits)? Monthly averages? Quarterly averages?

Weekly (7 days) averages: $0.70 \pm 2(0.123)(0.70) =$
0.70 ± 0.17,

Monthly (30 days) averages: $0.70 \pm 2(0.069)(0.70) =$
0.70 ± 0.10,

Quarterly (90 days) averages: $0.70 \pm 2(0.04)(0.70) =$
0.70 ± 0.056.

Example 2

Only two unit trains are received per week (eight per month), and the limits of variation specified in Example 1 are required. In this case one must refer to Table 5-1, third column. The results are as follows:

Weekly averages: $0.70 \pm 2(0.142)0.70 =$
0.70 ± 0.199,

Monthly averages: $0.70 \pm 2(0.078)0.70 =$
0.70 ± 0.109,

Quarterly averages: $0.70 \pm 2(0.045)0.70 =$
0.70 ± 0.063.

Example 3

Data are available from which it has been determined that monthly averages vary by 10 percent of the mean (RSD = 10%) about an average sulfur content of 0.70 weight percent and that the heating values vary by 2 percent about an average of 12,000 Btu/lb. Would this coal enable the plant to meet the NSPS regulation for 95 percent of the monthly averages? The average and standard deviation of lb SO_2/MM Btu is given by the very good approximation,

$$\bar{x} \approx \frac{0.70 \times 1.90}{12{,}000} \times 10^4 = 1.11,$$

$$s \approx \bar{x} \sqrt{(0.10)^2 + (0.02)^2} = 1.11(0.102) = 0.113.$$

Using Figure 5-5, for RSD of 10.2 percent and 95 percent compliance, the required average must not exceed 1.03 (1.01) lb SO_2/MM Btu assuming normal (lognormal) distribution. Because $\bar{x} = 1.11$ is larger than 1.03, the relative frequency of compliance will not be 95 percent. The expected frequency of compliance can be estimated, assuming normality, by use of the standard normal variable, u,

$$u = \frac{1.2 - \bar{x}}{s} = \frac{1.2 - 1.11}{0.113} = 0.80,$$

and a table of normal probabilities[8] to obtain 0.788, or compliance about 79 percent of the time.

Example 4

It has been determined that the variability of sulfur content (weight percent) of a unit train sample has an RSD equal to 15 percent. The average sulfur content required for compliance 95 percent of the time is about 0.96 (0.94) lb SO_2/ MM Btu, as read from Figure 5-5 for the assumption of normal (lognormal) distribution. Note that Figure 5-5 can be used in many applications.

Example 5

A state agency has decided that the monthly averages of sulfur content must provide compliance with the NSPS regulation 99 percent of the time. Assume that a plant receives four unit trains each week, or sixteen per month. What is the average sulfur content required for compliance with the regulation 1.2 lb SO_2/MM Btu? It is assumed that Table 5-1 is applicable, that is, the RSD of weight percent sulfur is 0.069 and the RSD for lb SO_2/MM Btu is given by 0.069 (1.05) = 0.0725.

From Figure 5-5, the 30-day average for 99 percent compliance and assuming normal distribution is read as 1.025 lb SO_2/MM Btu. From Figure 5-3, for heating value of 11,500 Btu/lb, we read 0.727 percent sulfur to yield 1.2 lb or 0.727 (1.025/1.2) = 0.62 as the required average weight percent sulfur. Figure 5-3 was derived on the assumption that 1.9 pounds of SO_2 is emitted per pound of sulfur.

Example 6

Same as in Example 5, except that weekly averages are used and 95 percent compliance is acceptable. Using Table 5-1, the RSD for weekly averages (four unit trains) is 12.3 percent; from the 95 percent curve on Figure 5.5 (lognormal distribution), the required average sulfur content should be 0.98 lb SO_2/MM Btu. From Figure 5-3, the average sulfur content in weight percent sulfur is determined by reading the value corresponding to 11,500 (i.e., 0.727) and multiplying by 0.98/1.2 to obtain 0.59 weight percent sulfur.

Example 7

An SIP requires compliance 95 percent of the time on emission standard of 2.0 lb SO_2/MM Btu, based on monthly averages of 16 unit trains. Assume that the RSD values in Table 5-1 are applicable. From the table, we read the RSD value 6.9 percent. For compliance 95 percent of the time, the standard normal variable $u = (2.0 - m)/0.069(1.05)m$ must equal 1.645 (2.326 for 99 percent compliance). The 1.05 multiplier is used to account for the higher variability of lb SO_2/MM Btu relative to the variability of percent sulfur. Solving the equation,

$$\frac{2.0 - m}{0.0725m} = 1.645,$$

we obtain $m = 1.787$ lb SO_2/MM Btu. From Figure 5-3, the required average weight percent sulfur is obtained for an assumed heating value of 12,500 Btu/lb: $0.790 \times (1.787/1.2) = 1.18$ percent.

6.0 CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

1. The distribution of weight percent sulfur for composite samples from unit trains is skewed to the right (has a long tail of large values). This is particularly important when decisions are based on only one or two composite samples from unit trains.

2. The logarithmic normal distribution is used to approximate the skewed distribution. This is a good empirical approximation and is convenient for making predictions concerning compliance. The inverted gamma distribution has also been proposed as an empirical fit to the skewed frequency distribution.[1] Differences in estimates made using these two distributions are very small.

3. The distribution of Btu content of coals is reasonably symmetrical and is approximated rather well using the normal distribution.

4. The distribution of lb SO_2/MM Btu is skewed because of the dominating effect of the distribution of weight percent sulfur.

5. The variation of weight percent sulfur from composite samples (unit trains) is approximately 15 percent of the mean sulfur content, with almost all of the values between 10 and 25 percent of the mean. The lower percentages result from coal cleaning and/or from mining techniques, such as the number of machines in operation. The higher values would occur with no cleaning or with a mining technique that does not tend to mix or "blend" the coals within

the mining area. Greater variation also results from mixing coals from two seams with different sulfur content, particularly when the proportions of the two coals also vary. NOTE: It is emphasized that the values provided in this report are not necessarily representative of a specific coal, mining or processing technique. The values are based on a collection of coal data made available by selected companies. Each company using the approach presented herein is urged to use its own data in estimating the variability of sulfur content for specified averaging times or tonnages.

6. The RSD of Btu content is smaller than that of sulfur content; that is, the RSD of Btu content is about 2 to 5 percent of the mean Btu content compared to the RSD of sulfur content that is 10 to 25 percent of the mean sulfur content.

7. The RSD of lb SO_2/MM Btu is about 1.02 to 1.05 times the RSD for weight percent sulfur because of the dominating effect of the variation of sulfur. This is not true for the absolute standard deviation (see Appendix B.1.5). Throughout this report a conservative estimate of the RSD of lb SO_2/MM Btu is 1.05 × RSD of weight percent sulfur.

8. The sulfur contents of samples from consecutive unit trains will differ by much less than the sulfur contents of samples from trains far apart in time; that is, there is a statistical dependence in the time series of data. Hence, the statistical analysis includes a subdivision of the variation into two components (1) that among unit trains within a week (or month) and (2) that among weekly (or monthly) averages. For a few data sets the year-to-year component can be estimated. These values are then used to determine the impact of averaging period/amount of coal sampled (week, month, quarter; 33 T, 600 T, etc.).

9. The average sulfur content required for compliance with the 1.2 lb SO_2/MM Btu standard must decrease as the amount of coal in tons and/or the averaging period decreases and as the percentage of compliance time (95%, 99%, etc.) increases. Theoretically, if 100 percent compliance is required, then the average sulfur content approaches zero.

10. The RSD of sulfur content of coal from core drillings is greater by a factor of 1-1/2 to 4 than the variability of sulfur content among unit trains. This indicates the impact of small-increment samples on sulfur variability, since the core sample is a composite sample from a very small amount of coal, approximately 50 pounds.

11. Although sulfur variability is expected to decrease as the stage at which the sample is taken proceeds from core drilling → run-of-mine coal → as-cleaned coal → as-received at plant → as-burned, the reduction is difficult to detect beyond the first stage core drillings to run-of-mine coal. The reduction of variability in the latter stages should be relatively small; considerable data would be required to verify an expected reduction, if any.

12. Given an estimate of the RSD of sulfur content (weight percent), Figure 5-5 provides a straightforward means of determining the sulfur content in lb SO_2/MM Btu required for compliance with the regulation of 1.2 lb SO_2/MM Btu.

13. A quality control chart is suggested as one means of monitoring the sulfur content of coal to det[illegible] significant trends or changes in either the average o[illegible]ation of sulfur content over time. Section B.4 of Appendix B discusses this approach, proposing either a standard chart for averages or one for moving averages.

14. The correlation of Btu content and sulfur content is relatively low and in almost all cases is insignificantly different from zero.

RECOMMENDATIONS

1. As further coal cleaning data become available, it would be desirable to assess the variability of sulfur content before and after cleaning.

2. Additional long-term data (over several years) must be analyzed to determine long-term contractual implications. Recall from Section 2 that for strip mining operations with one machine the long-term variations are expected to be large. Not enough data have been obtained to assess this potential impact with the low-sulfur coals.

3. A computer program incorporating the types of analyses proposed in this report would have long-term utility. Furthermore, it would enable the analysis of some of the remaining data available to EPA.

4. Further consideration should be given to impact of sulfur variability, meteorological conditions, and other factors on the environmental SO_2 concentrations.

5. A study of the cost impacts of schemes for reducing the average and/or standard deviation of sulfur content would be helpful.

6. Additional data relating the variability of sulfur content to the stages of sampling (core drillings, channel samples, run-of-mine, after cleaning, and as received at the plant) would yield estimates of the expected average reduction (if any) at each stage. In this report the analysis is limited to a comparison of the core drillings and as-received data.

7. An ideal program for further study would be based on composite samples from smaller amounts of coal than the tonnages of unit trains, e.g., 50, 250, 1000, 5000, and 10,000 tons. At least 15 to 25 samples at each level would be needed. Analysis of these data would provide a basis for validating or adjusting the extrapolated region of the curve in Figure 5-1.

8. A study of the correlation between the RSD of data collected by a continuous stack monitor and the RSD of the as-burned coal would enhance the understanding of the potential change in variability at this stage.

7.0 REFERENCES

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APPENDIX A

SELECTED DATA SETS

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DATA SET C-1	A-1
DATA SET C-2	A-2
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DATA SET C-5	A-7
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DATA SET C-11	A-10
DATA SET C-12	A-11
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DATA SET U-1	A-13
DATA SET U-2	A-18
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Data Set C-1

Variability of Sulfur Content versus Amount of Coal Sampled

Weight percent sulfur	Amount of coal sampled (1000 tons)				
	< 8	8 - 12	12 - 16	16 - 20	Total
2.1 - 2.2	0	2	0	0	2
2.2 - 2.3	2	2	0	0	4
2.3 - 2.4	4	12	1	1	18
2.4 - 2.5	4	29	3	6	42
2.5 - 2.6	3	36	3	15	57
2.6 - 2.7	4	48	12	35	99
2.7 - 2.8	9	67	11	39	126
2.8 - 2.9	5	66	9	41	121
2.9 - 3.0	10	68	7	24	109
3.0 - 3.1	5	52	6	5	68
3.1 - 3.2	1	24	0	9	34
3.2 - 3.3	3	4	1	3	11
3.3 - 3.4	0	1	1	4	6
3.4 - 3.5	1	1	0	0	2
3.5 - 3.6	0	0	0	0	0
3.6 - 3.7	1	1	0	1	3
Total	52	413	54	183	702
Mean	2.801	2.803	2.788	2.805	2.802
Standard deviation	0.301	0.226	0.199	0.200	0.224
RSD	0.107	0.081	0.072	0.072	0.080

DATA SET C-2

CROSS-TABULATION OF SULFUR AND BTU CONTENT

Btu	Weight Percent Sulfur									
	0.54-0.55	0.56-0.57	0.58-0.59	0.60-0.61	0.62-0.63	0.64-0.65	0.66-0.67	0.68-0.69	0.70-0.71	Subtota
< 12,000	2	1	0	0	0	0	0	0	0	3
12,000-12,100	1	1	1	3	0	0	0	0	0	6
12,100-12,200	0	1	1	7	7	4	0	0	1	21
12,200-12,300	2	2	4	5	9	2	1	1	0	26
12,300-12,400	0	3	9	9	5	3	2	0	0	30
12,400-12,500	0	1	1	8	3	1	0	0	0	14
12,500-12,600	1	0	0	4	3	4	0	0	0	12
12,600-12,700	0	0	0	1	0	0	0	0	0	1
12,700-12,800	0	0	0	0	1	0	0	0	1	2
Subtotals	6	9	15	37	28	14	3	1	2	115

Weight Percent Sulfur	Btu
$\bar{x}$ = 0.61 weight percent sulfur	$\bar{x}$ = 12,310 Btu
s = 0.03 weight percent sulfur	s = 158 Btu

ANALYSIS OF VARIANCE OF SULFUR CONTENT

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among months	0.03	11	0.0027
Within months	0.07	103	0.0007

Estimated Standard Deviation (within months) $= (0.0007)^{\frac{1}{2}}$
$= 0.026$

Estimated Standard Deviation (among months) $= \left(\frac{0.0027 - 0.0007}{9}\right)^{\frac{1}{2}}$
$= 0.015$

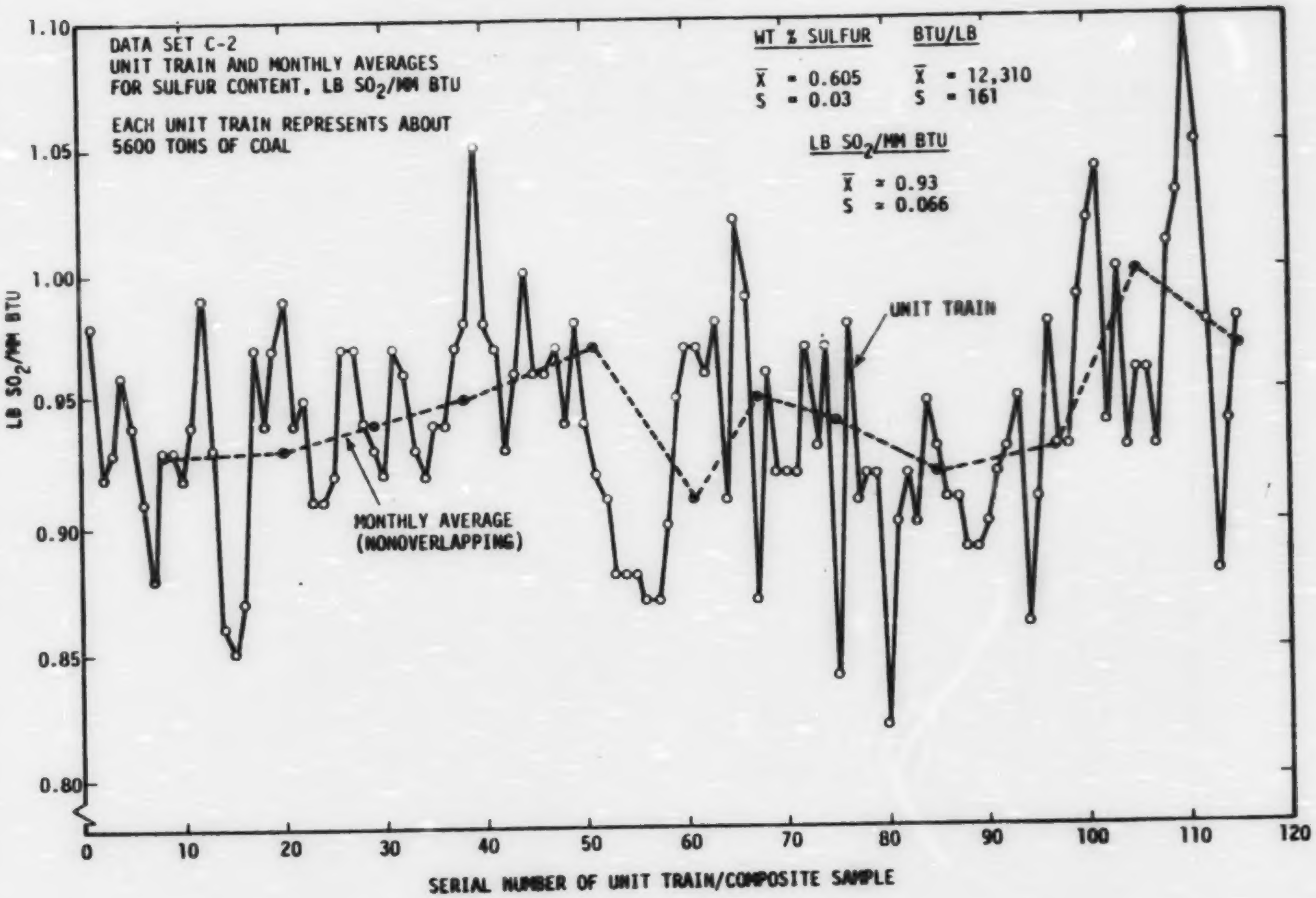

Figure A-1. Variation in sulfur content, lb SO_2/MM Btu with time, Data Set C-2.

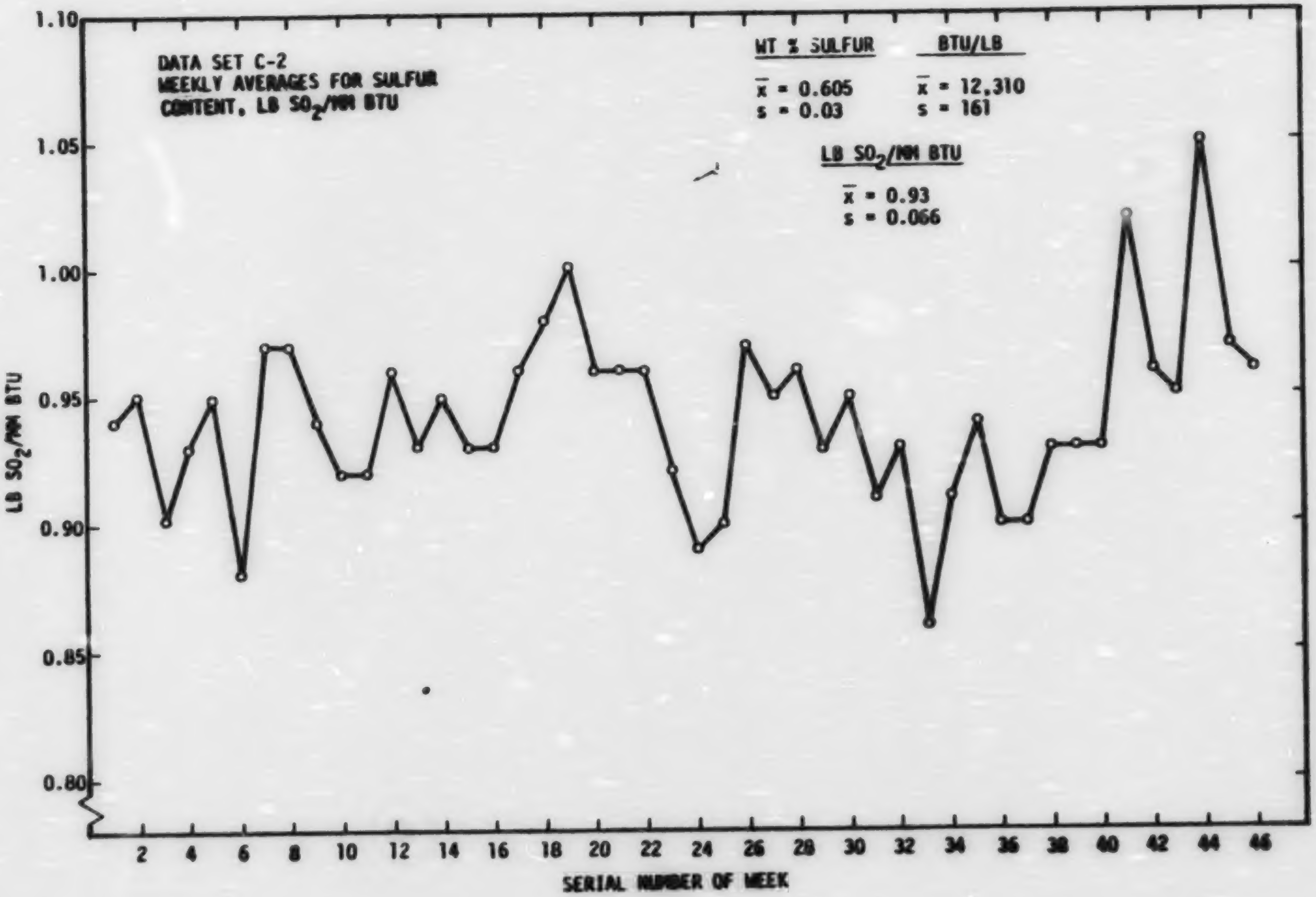

Figure A-2. Variation in weekly average sulfur content, lb SO_2/MM Btu.

DATA SET C-3

CROSS-TABULATION OF SULFUR AND BTU CONTENT

Btu	Weight Percent Sulfur													
	0.54-0.57	0.58-0.61	0.62-0.65	0.66-0.69	0.70-0.73	0.74-0.77	0.78-0.81	0.82-0.85	0.86-0.89	0.90-0.93	0.94-0.97	0.98-1.01	≥1.02	Sub
< 11,700	0	0	0	3	0	0	2	0	0	0	0	0	0	
11,700-11,800	0	0	0	0	4	1	1	0	0	0	0	0	0	
11,800-11,900	0	0	0	1	2	1	1	0	0	0	0	0	0	
11,900-12,000	1	0	0	2	3	5	2	1	0	0	0	0	0	
12,000-12,100	0	0	1	1	3	4	8	2	2	1	0	0	0	
12,100-12,200	0	0	1	1	1	2	4	5	2	0	0	0	0	
12,200-12,300	0	0	0	1	2	2	6	5	2	0	0	0	0	
12,300-12,400	0	0	0	0	5	4	5	4	3	0	1	0	0	
12,400-12,500	0	0	0	1	1	1	6	6	3	1	0	1	0	
12,500-12,600	1	0	0	0	1	0	2	4	3	0	0	0	1	
12,600-12,700	0	0	0	0	1	4	0	2	2	1	0	0	2	
12,700-12,800	0	0	0	0	0	0	1	0	1	1	0	0	0	
12,800-12,900	0	0	0	0	0	0	0	0	0	1	0	0	0	
12,900-13,000	0	0	0	0	0	0	0	0	0	0	0	0	0	
13,000-13,100	0	0	0	0	0	0	0	0	1	0	0	0	0	
13,100-13,200	0	0	0	0	0	0	0	0	0	0	0	0	0	
Subtotals	2	0	2	10	23	24	38	29	19	5	1	1	3	1

Weight Percent Sulfur	Btu
$\bar{x}$ = 0.807 weight percent sulfur	$\bar{x}$ = 12,250 Btu
s = 0.088 weight percent sulfur	s = 276 Btu

ANALYSIS OF VARIANCE OF SULFUR CONTENT

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among months	0.201	11	0.0182
Within months	0.841	146	0.0057

Estimated Standard Deviation (within months) $= (0.0057)^{\frac{1}{2}} = 0.076$

Estimated Standard Deviation (among months) $\simeq \left(\dfrac{0.0182 - 0.0057}{13}\right)^{\frac{1}{2}}$

$= 0.031$

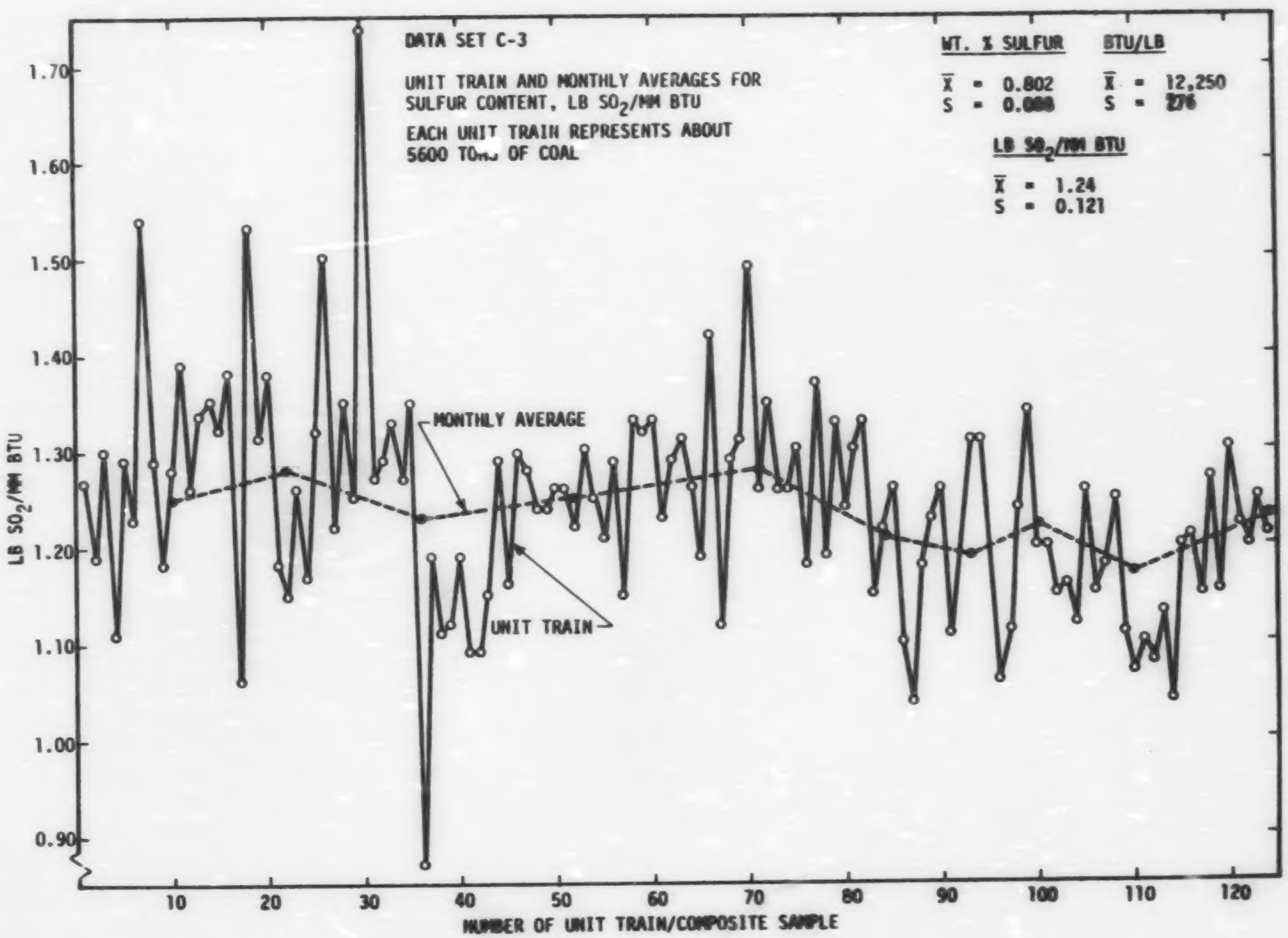

Figure A-3. Variation in sulfur content, lb SO_2/MM Btu, vs. time, Data Set C-3.

DATA SET C-5

CROSS-TABULATION OF SULFUR AND Btu VALUES

Btu	Weight Percent Sulfur							
	≤0.33	0.34-0.38	0.38-0.42	0.42-0.46	0.46-0.50	0.50-0.54	≥0.55	Total
11,000-11,200	0	0	0	0	0	0	0	0
11,200-12,400	0	0	0	1	0	0	0	1
11,400-11,600	0	0	0	1	0	1	0	2
11,600-11,800	0	1	0	6	2	2	0	11
11,800-12,000	0	1	11	12	9	5	7	45
12,000-12,200	0	18	37	55	6	5	2	123
12,200-12,400	5	42	27	30	10	2	0	116
12,400-12,600	1	10	2	4	0	0	0	17
12,600-12,800	0	0	0	0	0	0	0	0
12,800-13,000	0	0	0	1	0	0	0	1
Total	6	72	77	110	27	15	9	316

Sulfur	Btu
Mean, $\overline{x}$ = 0.42	Mean, $\overline{x}$ = 12,149
Standard Deviation, s = 0.0491	Standard Deviation, s = 197

DATA SET C-5

MEAN AND STANDARD DEVIATION (WEIGHT PERCENT)

(Based on a Sample of 10 Values Per Month)

Month	Mean			Standard Deviation		
	Year			Year		
	1974	1975	1976	1974	1975	1976
January	0.43	0.37	0.48	0.05	0.03	0.03
February	0.38	-	0.38	0.05	-	0.03
March	0.37	0.39	0.43	0.02	0.02	0.03
April	0.36	0.42	0.37	0.02	0.05	0.04
May	0.39	0.40	-	0.04	0.04	-
June	0.35	0.42	-	0.03	0.05	-
July	0.37	0.40	-	0.03	0.05	-
August	0.38	0.39	-	0.04	0.02	-
September	0.36	0.40	-	0.04	0.04	-
October	0.38	0.42	-	0.03	0.04	-
November	0.38	0.42	-	0.05	0.04	-
December	0.47	0.40	-	0.10	0.04	-

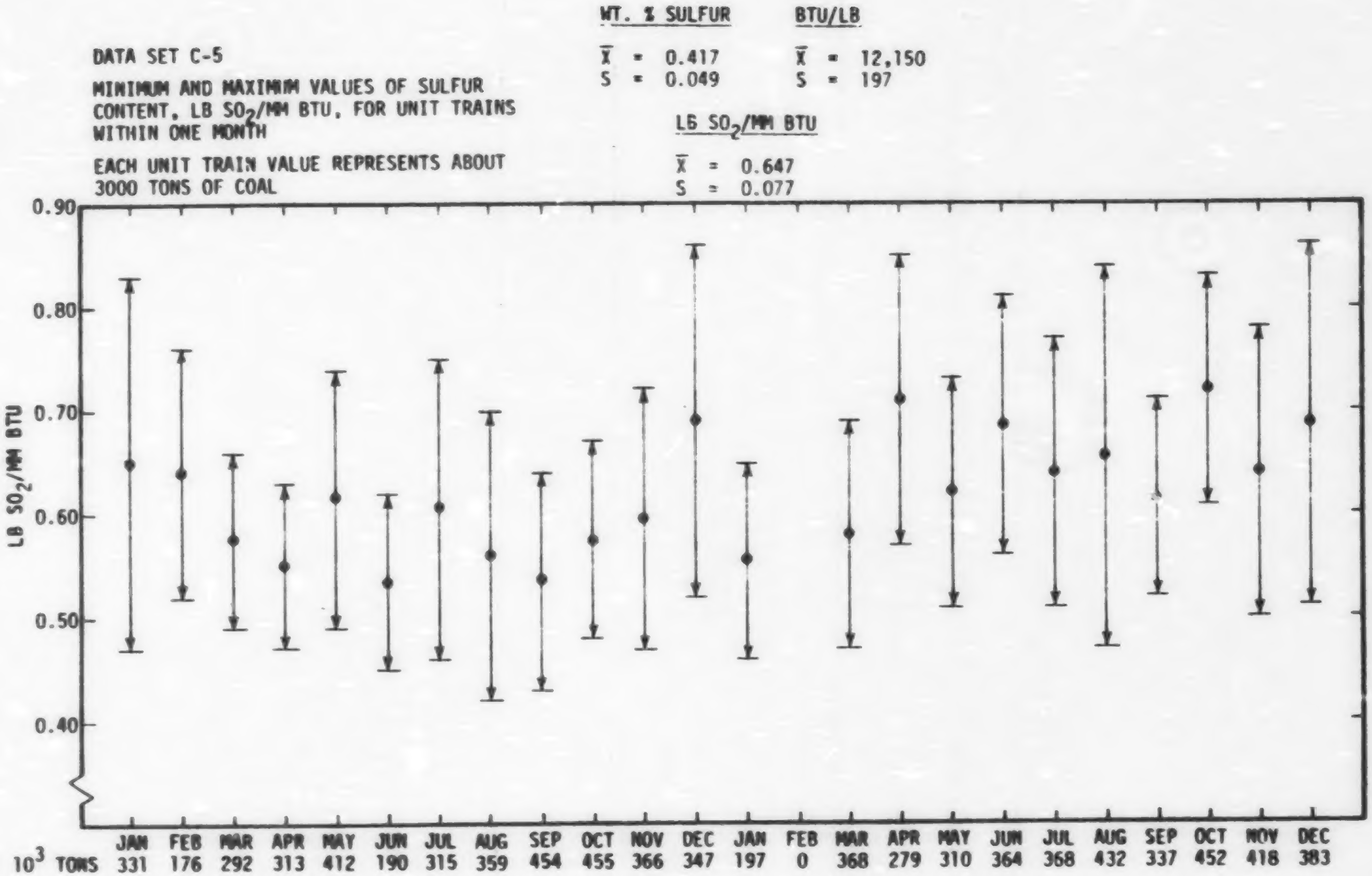

Figure A-4. Minimum and maximum values of sulfur content, lb SO_2/MM Btu, for unit trains within one month, Data Set C-5.

CORE DATA - TWO SEAMS

July 1976

Data Set C-10

Bottom Bench

	Minimum	Maximum	Mean	Variance	Standard Deviation
Sulfur	0.35	1.45	0.58	0.0612	0.2474
Btu	12134.00	13570.00	13174.63	109920.5625	331.5564
SO_2	0.52	2.39	0.88	0.1597	0.3996

Data Set C-11

Top Bench

	Minimum	Maximum	Mean	Variance	Standard Deviation
Sulfur	0.29	2.53	0.88	0.2858	0.5346
Btu	12042.00	13283.00	12685.27	84752.0625	291.1221
SO_2	0.44	3.99	1.39	0.7488	0.8653

SHIPMENT DATA - TWO SEAMS

July 1976

Data Set C-12

Sulfur	Minimum	Maximum	Mean	Variance	Standard deviation
By 2,000 tons	1.76	5.04	3.42	0.1807	0.4251
By 10,000 tons	2.54	4.34	3.42	0.1181	0.3436
By month	3.08	3.98	3.42	0.0682	0.2611
By year	3.18	3.54	3.42	0.0142	0.1193
Btu					
By 2,000 tons	9737.75	12719.95	12004.30	64764.4688	254.4886
By 10,000 tons	10964.39	12337.29	12003.53	25670.6094	160.2205
By month	11813.69	12227.21	12003.23	11930.5195	109.2269
By year	11935.29	12054.81	12003.22	3006.3992	54.8306
SO_2					
By 2,000 tons	3.00	9.52	5.69	0.5230	0.7232
By 10,000 tons	4.31	7.33	5.69	0.3451	0.5875
By month	5.09	6.68	5.69	0.1994	0.4466
By year	5.29	5.93	5.69	0.0468	0.2163

DATA SET C-13

CROSS-TABULATION OF SULFUR AND Btu VALUES

Btu	Weight Percent Sulfur										
	0.38-0.42	0.42-0.46	0.46-0.50	0.50-0.54	0.54-0.58	0.58-0.62	0.62-0.66	0.66-0.70	0.70-0.74	0.74-0.78	≥ 0.78	Tota
11,000-11,200	0	0	0	0	0	0	0	0	0	0	0	0
11,200-11,400	0	0	0	1	0	0	0	0	0	0	0	1
11,400-11,600	0	0	1	1	2	0	0	0	0	0	0	4
11,600-11,800	0	2	3	8	4	2	0	0	0	0	0	19
11,800-12,000	1	1	9	11	10	3	0	7	0	0	0	42
12,000-12,200	0	3	6	6	11	5	7	17	7	4	2	68
12,200-12,400	1	0	3	0	0	0	0	0	0	0	0	4
12,400-12,600	0	0	1	1	0	0	0	0	0	0	0	2
Total	2	6	23	28	27	10	7	24	7	4	2	140

Sulfur	Btu
Mean, $\overline{x}$ = 0.5768	Mean, $\overline{x}$ = 11,974
Standard Deviation, s = 0.0905	Standard Deviation, s = 191

DATA SET U-1

Date (1st day of wk)	Dry Basis			
	Weighted Weekly Average Sulfur	Weighted Weekly Average Btu	Weekly Range of Sulfur Content	Weekly Range of Btu Content
1973, May 31	0.72	11,512	0.32	068
June 7	0.76	12,465	0.84	1097
14	0.73	12,501	0.25	678
21	0.83	12,915	0.55	634
28	0.85	12,629	0.40	146
July 5	0.71	12,924	0.11	370
12	0.78	12,638	0.65	1217
19	0.74	12,530	0.17	1369
26	0.72	12,637	0.32	660
Aug 2	0.72	12,262	0.30	1867
9	0.70	11,730	0.16	1554
16	0.78	12,125	0.33	714
23	0.84	12,423	0.47	941
30	0.78	12,238	0.35	1069
Sept 6	0.88	12,591	0.56	456
13	0.94	12,730	0.46	808
20	0.99	12,726	1.05	1060
27	0.78	12,785	0.48	633
Oct 4	0.78	12,830	0.43	959
11	0.71	12,545	0.34	1800
18	0.70	12,535	0.29	1282
25	0.68	12,733	0.31	650
Nov 1	0.68	12,273	0.18	939
8	0.70	12,584	0.24	1041
15	0.64	12,566	0.12	580
22	0.93	12,583	0.94	913
29	0.76	11,562	0.70	342
Dec 6	0.74	12,765	0.36	931
13	0.68	12,815	0.27	123
20	0.65	12,787	0.26	1171
27	0.64	12,916	0.09	877
1974, Jan 3	0.68	12,643	0.20	817
10	0.64	12,679	0.08	947
17	0.62	12,499	0.25	1213
24	0.67	12,274	0.21	846
31	0.75	12,431	0.36	538
Feb 7	0.71	12,772	0.27	1102
14	0.70	12,648	0.37	469
21	0.74	12,549	0.32	411
28	0.77	12,726	0.39	494

DATA SET U-1 (continued)

Date (1st day of wk)	Dry Basis			
	Weighted Weekly Average Sulfur	Weighted Weekly Average Btu	Weekly Range of Sulfur Content	Weekly Range of Btu Content
Mar 7	0.78	12,615	0.66	1078
14	0.74	12,861	0.24	430
21	0.75	12,440	0.34	1173
28	0.68	12,700	0.24	800
Apr 4	0.70	12,876	0.14	1015
11	0.76	12,591	0.47	821
18	0.69	12,209	0.21	876
25	0.96	12,255	0.47	638
May 9	0.89	12,465	0.73	1326
16	0.81	12,725	0.24	960
23	0.84	12,367	0.23	756
30	0.85	12,517	0.40	607
June 6	0.89	12,407	0.58	676
13	0.80	12,704	0.14	343
20	0.69	12,333	0.69	1186
27	0.89	12,714	0.50	538
July 4	0.87	12,703	0.32	674
11	0.75	12,620	0.17	754
18	0.71	12,644	0.63	1002
25	0.66	12,049	0.09	724
Aug 1	0.70	12,192	0.07	686
8	0.73	12,445	0.26	476
15	0.71	12,482	0.16	861
22	0.76	12,266	0.30	470
29	0.73	12,121	0.26	1105
Sept 5	0.70	12,014	0.19	1089
12	0.68	12,166	0.18	532
19	0.70	12,330	0.14	623
26	0.72	12,852	0.09	621
Oct 3	0.67	12,464	0.07	672
10	0.63	12,203	0.10	1254
17	0.69	12,161	0.14	620
24	0.68	12,310	0.06	1320
31	0.66	12,756	0.09	566
Nov 7	0.72	12,634	0.15	567
21	0.71	12,687	0.20	503
28	0.70	12,815	0.18	242
Dec 5	0.77	12,705	0.34	416
12	0.70	12,318	0.15	448
19	0.68	12,304	0.11	922
26	0.72	12,268	0.18	427

DATA SET U-1 (continued)

Date (1st day of wk)	Dry Basis			
	Weighted Weekly Average Sulfur	Weighted Weekly Average Btu	Weekly Range of Sulfur Content	Weekly Range of Btu Content
1975, Jan 2	0.72	12,618	0.23	1065
9	0.64	12,646	0.20	411
16	0.70	12,686	0.14	680
23	0.67	12,399	0.09	838
30	0.71	12,279	0.08	837
Feb 6	0.69	12,292	0.11	635
13	0.81	12,519	0.21	781
20	0.71	12,282	0.09	854
27	0.73	12,484	0.20	1941
Mar 6	0.73	12,105	0.39	349
13	0.73	12,278	0.25	347
20	0.71	12,114	0.11	865
27	0.76	12,645	0.45	513
Apr 3	0.69	12,537	0.16	692
10	0.68	12,252	0.07	721
17	0.79	12,196	0.40	719
24	0.82	12,321	0.53	1368
May 1	0.79	12,110	0.36	1749
8	0.76	12,141	0.58	959
15	0.76	12,270	0.51	1122
22	0.77	12,705	0.53	1034
29	0.71	12,410	0.14	1061

DATA SET U-1 (continued)

FREQUENCY TABULATION OF WEEKLY WEIGHTED AVERAGE OF SULFUR - DRY BASIS

Weighted Average (%)	June - Dec 1973	Jan - June 1974	July - Dec 1974	Jan - May 1975	Total
.62 - .63	0	1	0	0	1
.64 - .65	3	1	1	1	6
.66 - .67	0	1	3	1	5
.68 - .69	3	4	4	3	14
.70 - .71	5	3	8	5	21
.72 - .73	4	0	5	4	13
.74 - .75	2	3	1	0	6
.76 - .77	2	3	2	4	11
.78 - .79	5	1	0	2	8
.80 - .81	0	2	0	1	3
.82 - .83	1	0	0	1	2
.84 - .85	2	2	0		4
.86 - .87	0	0	1		1
.88 - .89	1	3			4
.90 - .91	0	0			0
.92 - .93	1	0			1
.94 - .95	1	0			1
.96 - .97	0	1			1
.98 - .99	1	0			1
$\bar{x}$ s	0.762 0.0869	0.761 0.0868	0.711 0.0435	0.730 0.0470	0.742 0.0732

DATA SET U-1 (continued)

FREQUENCY TABULATION OF WEEKLY RANGE OF WEIGHT PERCENT SULFUR - DRY BASIS

Range (%)	June - Dec 1973	Jan - June 1974	July - Dec 1974	Jan - May 1975	Total
0 - .05	0	0	1	1	2
.06 - .10	1	1	6	4	12
.11 - .15	2	2	7	3	14
.16 - .20	4	4	6	2	16
.21 - .25	3	6	0	3	12
.26 - .30	2	1	3	1	7
.31 - .35	6	2	2	2	12
.36 - .40	2	4		1	7
.41 - .45	2	0		1	3
.46 - .50	2	3		0	5
.51 - .55	1	0		3	4
.56 - .60	0	1		1	2
.61 - .65	1	0			1
.66 - .70	2	0			2
.71 - .75	0	1			1
.76 - .80	0				0
.81 - .85	1				1
.86 - .90	0				0
.91 - .95	0				0
.96 - 1.0	1				1
1.01 - 1.05	1				1
$\bar{R}$	0.394	0.306	0.160	0.260	0.287

DATA SET U-2

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR
(As Received at Plant)

	Frequency for Year		
% Sulfur	1973	1974	1975
0.7 - 0.8	58	51	7
0.9 - 1.0	64	83	44
1.1 - 1.2	0	2	2
1.3 - 1.4	2	0	0
Total	124	136	53
Mean, $\bar{x}$ =	0.86	0.88	0.93
Standard Deviation, s =	0.12	0.10	0.08

ANALYSIS OF VARIANCE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among Months	0.54	27	0.0200
Within Months	0.58	284	0.0020

Estimated Standard Deviation (within months) $= (0.0020)^{\frac{1}{2}}$

$$= 0.045$$

Estimated Standard Deviation (among months) $\approx \left(\frac{0.0200 - 0.0020}{11}\right)^{\frac{1}{2}}$

$$= 0.040$$

DATA SET U-3

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR
(Coal Burned at Same Plant as for Data Set U-2)

% Sulfur	Frequency for Year			Total
	1973	1974	1975	
0.5 - 0.6	1	0	0	1
0.7 - 0.8	27	37	12	76
0.9 - 1.0	6	6	4	16
1.1 - 1.2	2	2	0	4
1.3 - 1.4	3	2	0	5
1.5 - 1.6	1	0	0	1
Total	40	47	16	103
Mean, $\bar{x}$ =	0.86	0.82	0.80	
Standard Deviation, s =	0.22	0.15	0.09	

ANALYSES OF VARIANCE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among Months	1.37	25	0.0548
Within Months	1.39	76	0.0183

Estimated Standard Deviation (within months) =

$$(0.0183)^{\frac{1}{2}} = 0.135$$

Estimated Standard Deviation (among months) $\approx \left(\frac{0.0548 - 0.0183}{4}\right)^{\frac{1}{2}}$

$$= 0.096$$

DATA SET U-4

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR
(As received at plant from various seams of coal - 1 county)

% Sulfur	Frequency Distribution by Year			
	1974	1975	1976	Total
0.7 - 0.8	6	1	0	7
0.9 - 1.0	34	49	22	105
1.1 - 1.2	6	27	5	38
1.3 - 1.4	2	7	0	9
1.5 - 1.6	1	2	0	3
1.7 - 1.8	0	0	0	0
1.9 - 2.0	0	1	0	1
≥ 2.1	1 (4.1)	0	0	1 (4.1)
Total	50	87	27	164
Mean, $\overline{x}$ =	1.04 (0.98)*	1.07	0.99	
Standard Deviation, s =	0.47 (0.15)*	0.18	0.08	

*Omitting 4.1 value

DATA SET U-4

FREQUENCY DISTRIBUTION OF Btu VALUES

	Year			
Btu	1974	1975	1976	Total
11,200 - 11,400	0	3	1	4
11,400 - 11,600	0	5	2	7
11,600 - 11,800	9	18	5	32
11,800 - 12,000	9	19	10	38
12,000 - 12,200	13	24	2	39
12,200 - 12,400	8	16	4	28
12,400 - 12,600	3	5	3	11
12,600 - 12,800	2	1	1	4
12,800 - 13,000	0	1	0	1
	44	92	28	164
Mean, $\overline{x}$ =	12,068	11,993	12,014	
Standard Deviation, s =	276	310	319	

DATA SET U-4

FREQUENCY DISTRIBUTION OF lbs SO_2/MM Btu

(All Years, 1974 - 1976)

lbs SO_2/MM Btu	Frequency
1.1 - 1.2	3
1.3 - 1.4	7
1.5 - 1.6	58
1.7 - 1.8	58
1.9 - 2.0	22
2.1 - 2.2	5
2.3 - 2.4	6
2.5 - 2.6	1
2.7 - 2.8	1
2.9 - 3.0	0
3.1 - 3.2	0
3.3 - 3.4	1
Total	162

$\overline{x}$ = 1.73

s = 0.28

DATA SET U-5

FREQUENCY DISTRIBUTION OF Btu VALUES

	Year			
Btu	1974	1975	1976	Total
11,000 - 11,200	0	2	0	2
11,200 - 11,400	2	12	0	14
11,400 - 11,600	3	22	4	29
11,600 - 11,800	8	40	5	53
11,800 - 12,000	19	43	5	67
12,000 - 12,200	24	26	5	55
12,200 - 12,400	9	10	2	21
12,400 - 12,600	2	4	0	6
12,600 - 12,800	3	0	0	3
Total	70	159	21	250

Mean, $\bar{x}$ = 12,014 11,812 11,862

Standard Deviation, s = 286 293 258

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR

(As received at plant from various seams of coal - 1 county)

Frequency Distribution by Year

% Sulfur	1974	1975	1976	Total
0.7 - 0.8	28	34	1	63
0.9 - 1.0	42	110	15	167
1.1 - 1.2	0	14	4	18
> 1.3	0	1 (2.2)	1 (2.1)	2
Total	70	159	21	250

Mean, $\bar{x}$ = 0.87 0.93 (0.925)* 1.03 (0.98)**

Standard Deviation, s = 0.10 0.15 (0.11)* 0.26 (0.10)**

* Omitting 2.2 value. **Omitting 2.1 value.

DATA SET U-5

FREQUENCY DISTRIBUTION OF lbs SO_2/MM Btu
(All Years, 1974 - 1976)

lbs SO_2/MM Btu	Frequency
1.1 - 1.2	9
1.3 - 1.4	56
1.5 - 1.6	125
1.7 - 1.8	44
1.9 - 2.0	14
$\geq$ 2.1	2
	Total = 250

$\bar{x} = 1.55$

$s = 0.18$

ANALYSIS OF VARIANCE OF SULFUR CONTENT

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among months	1.48	22	0.067
Within months	3.94	227	0.017

Estimated Standard Deviation (within months) $= (0.017)^{\frac{1}{2}}$

$= 0.132$

Estimated Standard Deviation (among months) $= \left(\frac{0.067 - 0.017}{10.4}\right)^{\frac{1}{2}}$

$= 0.069$

DATA SET U-6

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR
(As Received at Plant)

% Sulfur	Frequency
0.6	22
0.7	18
0.8	21
0.9	12
1.0	13
1.1	4
Total	90

$\overline{x}$ = 0.79

s = 0.15

FREQUENCY DISTRIBUTION OF BTU

Btu	Frequency
10,600-10,700	4
11,000-11,100	3
11,100-11,200	0
11,200-11,300	5
11,300-11,400	0
11,400-11,500	0
11,500-11,600	8
11,600-11,700	13
11,700-11,800	8
11,800-11,900	17
11,900-12,000	2
12,000-12,100	22
12,100-12,200	0
12,200-12,300	4
12,300-12,400	0
12,400-12,500	0
12,500-12,600	4
Total	90

$\overline{x}$ = 11,770

s = 393

DATA SET U-6

FREQUENCY DISTRIBUTION OF lbs SO_2/MM Btu

lbs SO_2/MM Btu	Frequency
1.0	21
1.1	4
1.2	12
1.3	16
1.4	7
1.5	7
1.6	6
1.7	9
1.8	5
1.9	3
	Total = 90

$\overline{x}$ = 1.34

s = 0.27

ANALYSIS OF VARIANCE OF SULFUR CONTENT

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among months	0.71	13	0.055
Within months	1.33	76	0.018

$$\text{Estimated Standard Deviation (within months)} = (0.018)^{\frac{1}{2}} = 0.134$$

$$\text{Estimated Standard Deviation (among months)} \simeq \left(\frac{0.055 - 0.018}{7}\right)^{\frac{1}{2}} = 0.073$$

DATA SET U-6

CROSS-TABULATION OF SULFUR AND BTU CONTENT

Btu	Weight Percent Sulfur					
	0.6	0.7	0.8	0.9	1.0	1.1
10,500-10,600	0	3	1	0	0	0
10,900-11,000	1	0	0	0	2	0
11,000-11,100	0	0	0	0	0	0
11,100-11,200	0	0	0	3	2	0
11,200-11,300	0	0	0	0	0	0
11,300-11,400	0	0	0	0	0	0
11,400-11,500	0	2	0	3	0	3
11,500-11,600	0	2	7	0	4	0
11,600-11,700	3	0	0	4	2	0
11,700-11,800	7	5	2	0	3	0
11,800-11,900	0	0	2	0	0	0
11,900-12,000	6	3	9	2	0	1
12,000-12,100	0	0	0	0	0	0
12,100-12,200	4	0	0	0	0	0
12,200-12,300	0	0	0	0	0	0
12,300-12,400	0	0	0	0	0	0
12,400-12,500	1	3	0	0	0	0
Total	22	18	21	12	13	4

DATA SET U-7

FREQUENCY DISTRIBUTION OF Btu VALUES
(1974 Data)

Btu	Frequency
11,400 - 11,600	7
11,600 - 11,800	12
11,800 - 12,000	19
12,000 - 12,200	15
12,200 - 12,400	24
12,400 - 12,600	19
12,600 - 12,800	0
12,800 - 13,000	22
≥ 14,200	2
	Total 120

Mean, $\bar{x}$ = 12,278

Standard Deviation, s = 488

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR
(As Received at Plant)

% Sulfur	Frequency
0.6	21
0.7	38
0.8	23
0.9	6
1.0	30
≥ 1.5	2
	Total 120

Mean, $\bar{x}$ = 0.80

Standard
Deviation, s = 0.17

DATA SET U-7

FREQUENCY DISTRIBUTION OF lbs SO_2/MM Btu

lbs SO_2/MM Btu	Frequency
0.9 - 1.0	23
1.1 - 1.2	44
1.3 - 1.4	16
1.5 - 1.6	24
1.7 - 1.8	11
$\geq$ 2.3	2
	Total 120

Mean, $\overline{x}$ = 1.29

Standard Deviation, s = 0.28

ANALYSIS OF VARIANCE OF SULFUR CONTENT

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among months	0.51	11	0.046
Within months	2.95	108	0.027

Estimated Standard Deviation (within months) $= (0.027)^{\frac{1}{2}}$

$= 0.165$

Estimated Standard Deviation (among months) $\simeq \left(\frac{0.046 - 0.027}{10}\right)^{\frac{1}{2}}$

$= 0.0436$

DATA SET U-7

CROSS-TABULATION OF SULFUR AND Btu CONTENT

Btu	Weight Percent Sulfur						
	0.6	0.7	0.8	0.9	1.0	> 1.5	Total
11,500	1	0	0	0	0	0	
11,600	0	0	0	0	6	0	1
11,700	0	3	0	0	0	0	6
11,800	0	9	0	0	0	0	3
11,900	0	0	0	0	0	0	9
12,000	0	3	12	1	3	0	0
12,100	0	0	0	4	10	0	19
12,200	0	0	0	0	1	0	14
12,300	0	0	0	0	5	0	1
12,400	0	16	3	0	0	0	5
12,500	10	4	0	0	4	0	19
12,600	0	1	0	0	0	0	18
12,700	0	0	0	0	0	0	1
12,800	0	0	0	0	0	0	0
12,900	0	0	0	0	0	0	0
13,000	10	0	8	1	1	2	22
14,200	0	2	0	0	0	0	2
Total	21	38	23	6	30	2	120

DATA SET U-8

FREQUENCY DISTRIBUTION OF lbs SO_2/MM Btu

lbs SO_2/MM Btu	Frequency
0.9 - 1.0	7
1.1 - 1.2	22
1.3 - 1.4	11
1.5 - 1.6	10
1.7 - 1.8	7
$\geq$ 2.3	4
	Total 61

Mean, $\bar{x} = 1.37$

Standard Deviation, $s = 0.34$

ANALYSIS OF VARIANCE OF SULFUR CONTENT

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among months	0.068	8	0.0085
Within months	2.758	51	0.0541

Estimated Standard Deviation (within months) $= (0.0541)^{1/2}$

$= 0.233$

Estimated Standard Deviation (among months) $= \left(\dfrac{0.0085 - 0.0541}{60/9}\right)^{1/2} < 0$

$\approx 0^+$

Occasionally the estimated component of variance is negative due to chance variation. In these cases the component is shown as a 0^+ value in the report.

DATA SET U-8

FREQUENCY DISTRIBUTION OF Btu VALUES

Btu	Frequency
11,400 - 11,600	7
11,600 - 11,800	11
11,800 - 12,000	14
12,000 - 12,200	2
12,200 - 12,400	1
12,400 - 12,600	7
12,600 - 12,800	0
12,800 - 13,000	16
≥ 14,200	2
	Total 60

Mean, $\overline{x}$ = 12,243

Standard Deviation, s = 636

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR
(As Received at Plant)

% Sulfur	Frequency
0.6	5
0.7	22
0.8	13
0.9	3
1.0	13
≥ 1.5	4
	Total 60

Mean, $\overline{x}$ = 0.84

Standard
Deviation, s = 0.22

DATA SET U-8

CROSS-TABULATION OF SULFUR AND Btu CONTENT

Btu	Weight Percent Sulfur						
	0.6	0.7	0.8	0.9	1.0	1.5	Total
11,400-11,600	2	0	1	0	4	0	7
11,600-11,800	0	11	0	0	0	0	11
11,800-12,000	0	4	7	1	2	0	14
12,000-12,200	0	0	0	2	0	0	2
12,200-12,400	0	0	1	0	0	0	1
12,400-12,600	0	5	0	0	2	0	7
12,600-12,800	0	0	0	0	0	0	0
12,800-13,000	3	0	4	0	5	4	16
≥ 14,200	0	2	0	0	0	0	2
Total	5	22	13	3	13	4	60

DATA SET U-9

FREQUENCY DISTRIBUTION OF lbs SO_2/MM Btu

lbs SO_2/MM Btu	Frequency
0.9 - 1.0	8
1.1 - 1.2	23
1.3 - 1.4	19
1.5 - 1.6	15
1.7 - 1.8	7
1.9 - 2.0	1
$\geq$ 2.3	1
	Total 74

Mean, $\overline{x} = 1.35$

Standard Deviation, $s = 0.27$

ANALYSIS OF VARIANCE OF SULFUR CONTENT

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among months	0.12	11	0.011
Within months	1.65	62	0.027

Estimated Standard Deviation (within months) $= (0.027)^{\frac{1}{2}}$

$= 0.163$

Estimated Standard Deviation (among months) $= \left(\frac{0.011 - 0.027}{74/12}\right)^{\frac{1}{2}} < 0$

≈ 0[a]

---

[a] The estimated standard deviation among monthly averages is taken as zero in this particular case in which the estimated variance component is negative.

DATA SET U-9

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR

% Sulfur	Frequency
0.6	9
0.7	18
0.8	24
0.9	6
1.0	15
1.1	1
≥ 1.5	1
	Total 74

Mean, $\overline{x}$ = 0.81

Standard
Deviation, s = 0.16

FREQUENCY DISTRIBUTION OF Btu VALUES

Btu	Frequency
≤ 10,600	1
11,000-11,200	8
11,200-11,400	0
11,400-11,600	12
11,600-11,800	11
11,800-12,000	18
12,000-12,200	1
12,200-12,400	1
12,400-12,600	3
12,600-12,800	2
12,800-13,000	16
≥ 14,200	1
	Total 74

Mean, $\overline{x}$ = 12,003

Standard
Deviation, s = 665

DATA SET U-9

CROSS-TABULATION OF SULFUR AND Btu CONTENT

Btu	Weight Percent Sulfur							
	0.6	0.7	0.8	0.9	1.0	1.1	1.5	Total
≤ 10,600	0	0	1	0	0	0	0	1
11,000-11,200	2	0	0	4	0	0	0	8
11,200-11,400	0	0	0	0	0	0	0	0
11,400-11,600	0	0	8	0	3	1	0	12
11,600-11,800	0	11	0	0	0	0	0	11
11,800-12,000	0	4	11	1	2	0	0	18
12,000-12,200	0	0	0	1	0	0	0	1
12,200-12,400	0	0	0	0	1	0	0	1
12,400-12,600	0	2	0	0	1	0	0	3
12,600-12,800	0	0	0	0	2	0	0	2
12,800-13,000	7	0	4	0	4	0	1	16
≥ 14,200	0	1	0	0	0	0	0	1
Total	9	18	24	6	15	1	1	74

DATA SET U-11

FREQUENCY DISTRIBUTION OF WEIGHT PERCENT SULFUR
(As Received at Plant)

% Sulfur	Frequency
2.0-2.1	1
2.1-2.2	1
2.2-2.3	0
2.3-2.4	6
2.4-2.5	5
2.5-2.6	9
2.6-2.7	15
2.7-2.8	23
2.8-2.9	27
2.9-3.0	20
3.0-3.1	18
3.1-3.2	22
3.2-3.3	12
3.3-3.4	23
3.4-3.5	27
3.5-3.6	16
3.6-3.7	8
3.7-3.8	6
3.8-3.9	1
	Total 240 $\bar{x} = 3.1$ $s = 0.366$

DATA SETS U-14 to U-21

Data Set	N	Weight Percent Sulfur Mean, $\bar{x}$	Weight Percent Sulfur Standard Deviation, s
U-14	82	0.49	0.19
*U-15	83	0.53	0.16
U-16	395	0.68	0.18
U-17	132	0.66	0.18
**U-18	770	0.37	0.035
*U-19	113	0.46	0.055
U-20	768	0.60	0.092
U-21	1766	0.55	0.086

*The cumulative frequency functions for these data are given in the following graphs.

**The histogram is given in addition to the cumulative frequency graph.

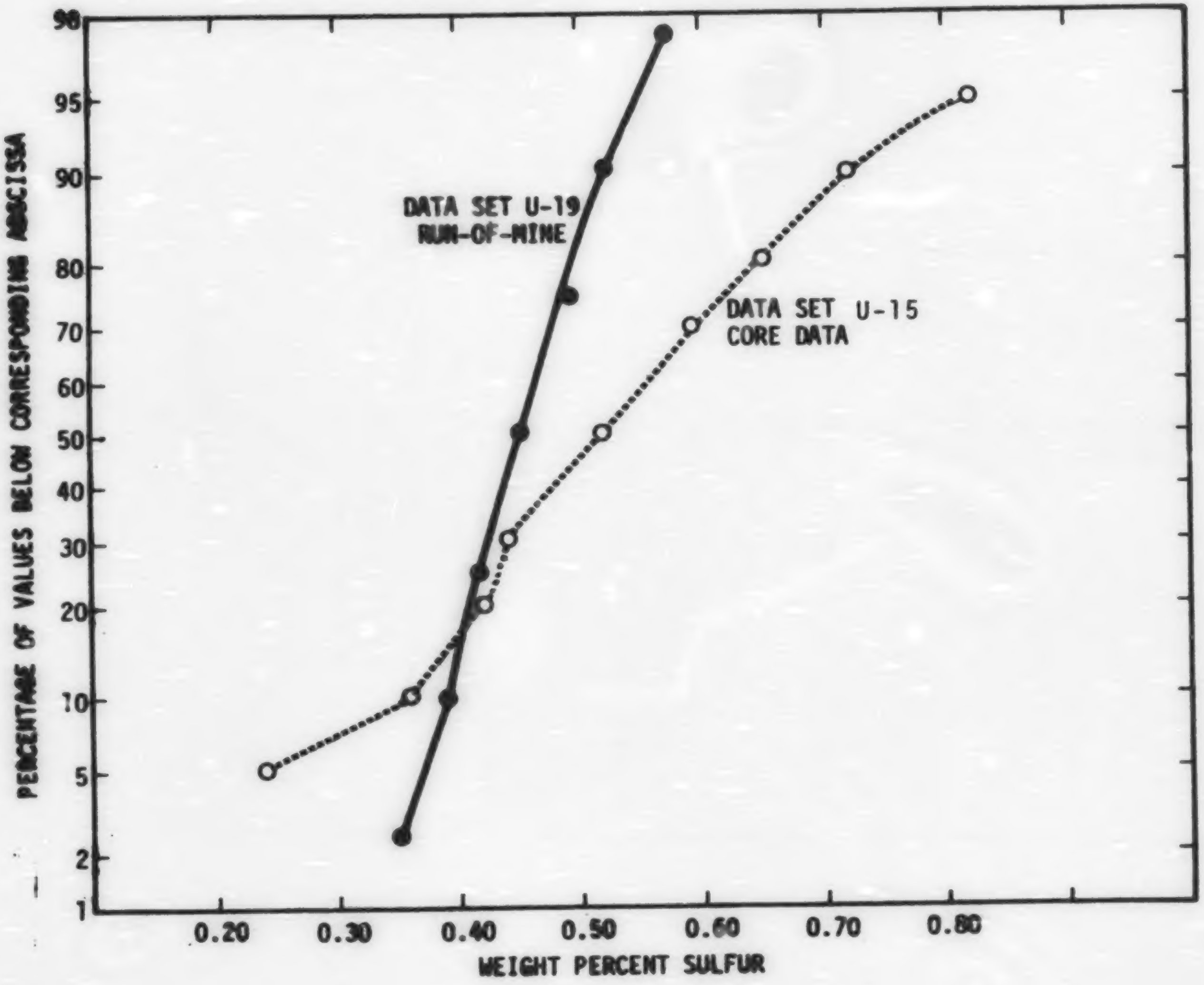

Figure A-5. Comparison of core data and run-of-mine data.

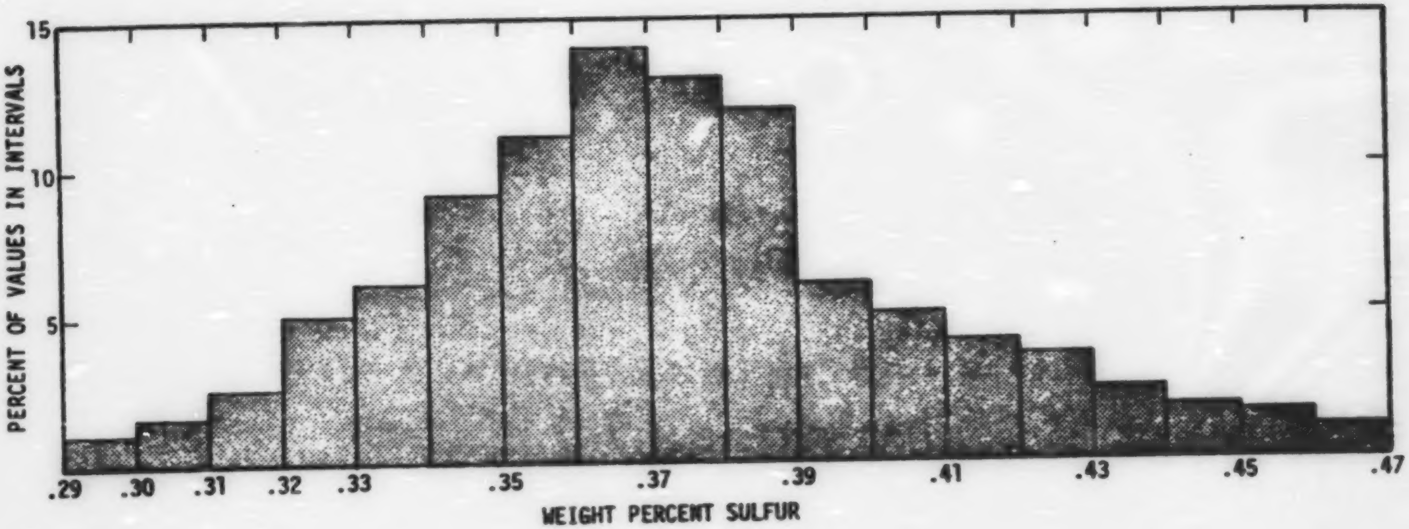

Figure A-6. Frequency distribution of one day averages of weight percent sulfur in coal. (Delivery data for Data Set U-18)

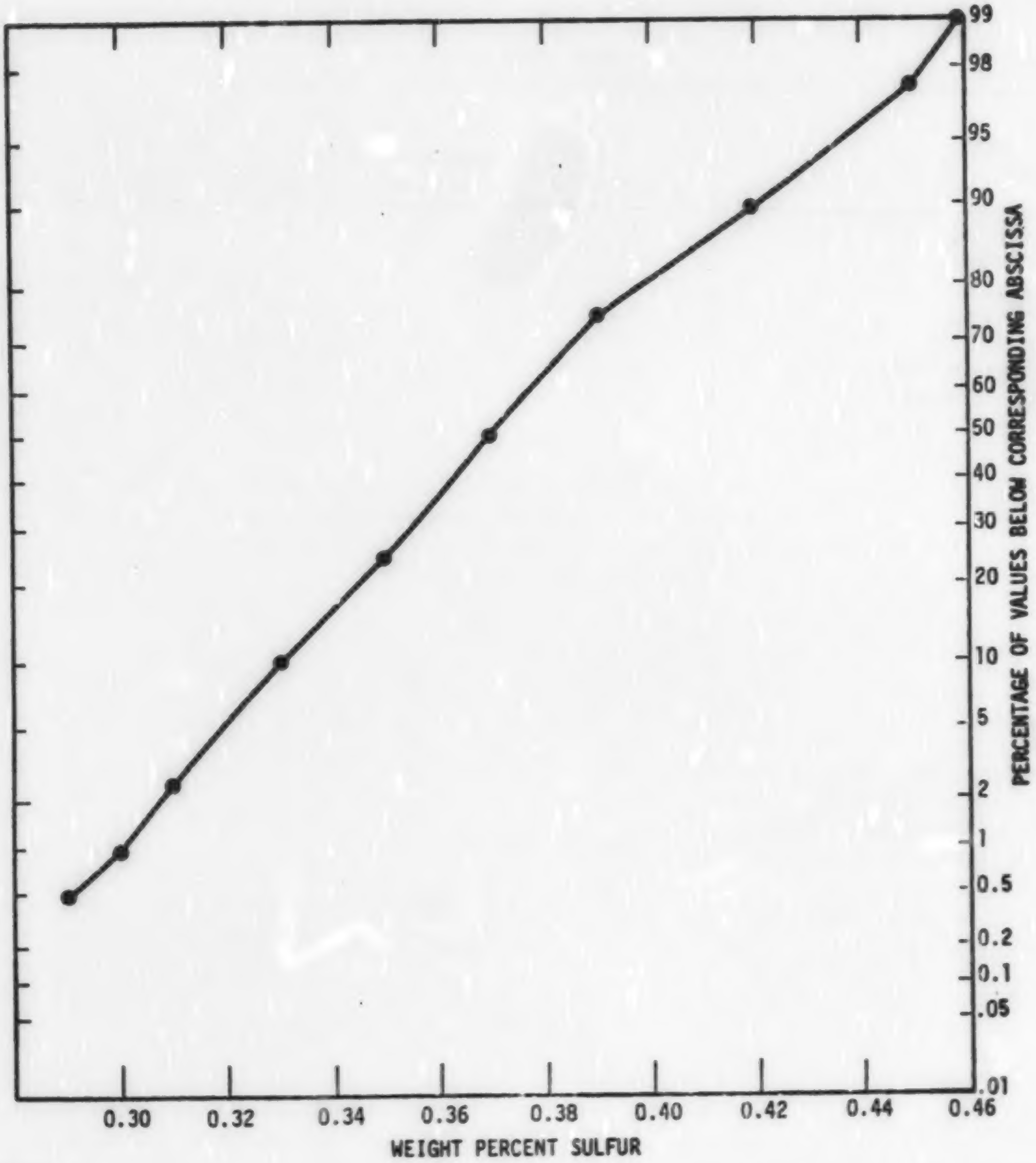

Figure A-7. Frequency distribution for Data Set U-18.

DATA SET U-22

			lbs SO_2/MM Btu		
Plant Name	District	Number of Analyses	Mean	Standard Deviation	RSD
Benning, D.C.	0354	19	1.06	0.21	20%
Winnetka, Ill.	0854	6	1.09	0.04	4%
Wood River, Ill.	0821	5	1.10	0.11	10%
Perry, Ind.	0821	2	1.16	0.03	3%
Smith, Md.	0354	13	0.98	0.13	13%
Advance, Mich.	0821	4	1.03	0.08	8%
Advance, Mich.	0854	2	1.17	0.00	0
Plant #65, Mich.	0851	2	1.09	0.15	14%
Missouri Ave., N.J.	0354	8	0.97	0.20	21%
Huntley, N.Y.	0854	2	0.98	0.05	5%
Group 1 - Pooled Estimates			1.03	0.16[a]	15.8%[b]
Stout, Ind.	0821	4	1.36	0.19	14%
Dickerson, Md.	0821	3	1.85	0.01	1%
Smith, Md.	0851	3	1.42	0.16	11%
Salem Harbor, Mass.	0821	14	1.35	0.32	24%
Marysville, Mich.	0751	2	1.24	0.10	8%
Trenton Channel, Mich.	0751	2	1.33	0.23	17%
Burlington, N.J.	0751	4	1.36	0.17	13%
Hudson, N.J.	0854	4	1.33	0.33	25%
Kearney A & B, N.J.	0751	2	1.37	0.22	16%
Mercer, N.J.	0751	6	1.36	0.16	12%
Cape Fear, N.C.	0851	72	1.42	0.18	5%
Cape Fear, N.C.	0854	32	1.39	0.24	17%
Cliffside, N.C.	0821	13	1.58	0.23	15%
Dan River, N.C.	0751	33	1.53	0.18	12%
Tait, Ohio	0854	28	1.35	0.22	16%
Titus, La.	0821	58	1.26	0.25	20%
Group 2 - Pooled Estimates			1.39	0.20[a]	14.6%[b]

[a] This standard deviation was obtained by weighting the corresponding variances by the number of degrees of freedom (number of analyses less one), dividing by the sum of the weights and then taking the square root.

[b] The RSD is computed from the ratio of the standard deviation to the mean, multiplied by 100 to convert to percent.

Table A-1. MINIMUM, MAXIMUM AND AVERAGE SULFUR WEIGHT PERCENT FOR EACH DATA SET

	Weight Percent Sulfur		
Data set	Minimum	Average	Maximum
C-1	2.10	2.80	3.70
C-2	0.54	0.60	0.71
C-3	0.54	0.80	1.02
C-4	0.65	1.38	3.58
C-5	0.30	0.42	0.56
C-6	0.89	2.14	3.97
C-7	0.75	1.50	3.55
C-8	2.22	2.60	3.19
C-9	0.60	1.63	4.90
C-10	0.35	0.58	1.45
C-11	0.29	0.88	2.53
C-12	2.54	3.42	4.34
C-13	0.38	0.58	0.80
U-1	0.62	0.74	0.99
U-2	0.70	0.88	1.40
U-3	0.50	0.82	1.60
U-4	0.70	1.04	2.10
U-5	0.70	0.92	1.30
U-6	0.60	0.79	1.10
U-7	0.60	0.80	1.50
U-8	0.60	1.04	2.30
U-9	0.60	0.81	1.50
U-10	0.65	1.74	3.00
U-11	2.00	3.10	3.90
U-12	1.72	2.25	2.85
U-13	1.64	2.15	2.63

APPENDIX B

METHODS OF DATA ANALYSES

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B.1 BASIC STATISTICAL ANALYSES

This appendix presents the statistical methods used in data analyses in this report. If the methods are not those routinely used, some discussion is presented along with the computational methods. The organization of the appendix permits the reader to look at only those sections for which he believes that further explanation would be helpful. It is assumed that the reader interested in further detail will refer to recommended references.

B.1.1 Mean, Standard Deviation, and Relative Standard Deviation of a Measurement Tabulated in a Frequency Table

Refer to Data Set U-1, January to May 1975 data on weight percent sulfur. The frequency tabulation is repeated from Appendix A for convenience. The midpoints of the intervals are added to the table to aid in the computation.

Weighted weekly average percent sulfur	Midpoint (x)	Frequency (f)
0.62 - 0.63	0.625	0
0.64 - 0.65	0.645	1
0.66 - 0.67	0.665	1
0.68 - 0.69	0.685	3
0.70 - 0.71	0.705	5
0.72 - 0.73	0.725	4
0.74 - 0.75	0.745	0
0.76 - 0.77	0.765	4
0.78 - 0.79	0.785	2
0.80 - 0.81	0.805	1
0.82 - 0.83	0.825	1

The following statistics are computed:

Average sulfur content = $\bar{x} = \frac{\Sigma fx}{\Sigma f}$ = 0.730 weight percent

Standard deviation of sulfur content =

$$s = \left[\frac{\Sigma f(x - \bar{x})^2}{n - 1}\right]^{1/2} = 0.0470,$$

and for ease of computation without a preprogrammed calculator

$$s = \left[\frac{\Sigma fx^2 - \{(\Sigma fx)^2/n\}}{n - 1}\right]^{1/2} .$$

Coefficient of variation (CV) or relative standard deviation (RSD)

$$\text{CV or RSD} = \frac{s}{\bar{x}} \times 100 = \frac{0.047}{0.730} \times 100 = 6.4\%.$$

The RSD is a very convenient measure of variation whenever the standard deviation tends to be proportional to the average value, see Section B.1.6. It is also useful in estimating the variance of ratios and products of variables, as illustrated in Appendix B.1.5. Three references are recommended, in order of increasing mathematical complexity.[9,5,6]

B.1.2 Analysis of Variance (Components of Variance)

This section discusses briefly the method of analysis of variance, including components of variance, the results of which are used in Section 5 to estimate the impact of averaging time on the average sulfur content required for compliance with NSPS.

The nested model with random components is assumed in all these analyses. An analysis of variance was conducted on several data sets to estimate the components of variation due to the variation of sulfur content among samples (unit trains) within the same week (or month) and among weekly (or monthly) averages. Data Set U-1 is used here to illustrate the computational approach. These data are weekly weighted averages (seven samples per week). Four consecutive weeks are treated as a month, giving actually a 28-day average. The analysis method is given in Table B-1, followed by an example in Table B-2. The weekly averages in Data Set U-1 are denoted by x and the totals of four consecutive values by T. Thus, there are 25 groups of 4 weeks if the last 3 weeks are eliminated. The analysis is straightforward, requiring only the sum and sum of squares of the individual weekly averages and the "monthly" totals. The individual computations for the example are given below and these are substituted in the formulas of Table B-1 to obtain the results tabulated in Table B-2.

$\Sigma x = 74.13$,

$\Sigma x^2 = 55.5107$, and

$\Sigma T^2 = 220.9187$.

The analysis follows methods given in standard texts such as reference 7. The sum of squares and mean squares are intermediate steps in the calculation and are used as a convenience to yield the mean square and the estimate of the components of variance. The mean square for "among weeks" within a month is a direct measure of the variance of the individual weekly averages about their own monthly average. The mean square for "among monthly" averages is an estimate of the variance of the weekly averages *plus* 4 times the variance of the monthly averages about an overall mean for *all* of the data. Equating the results in Table B-2, the estimates of the components of variance are obtained and given at the bottom of Table B-2.

Table B-1. ANALYSIS OF VARIANCE

Source of variation	Sum of squares (SS)	Degrees of freedom (DF)	Mean[a] square (MS)	Mean square estimate of indicated variance components
Among "monthly averages"	$\frac{\Sigma T^2}{4} - \frac{(\Sigma x)^2}{100}$	25 - 1 = 24	$\frac{SS}{24}$	Var (weekly average) + (4 x Var (monthly average))
Among weekly averages within month	$\Sigma x^2 - \frac{\Sigma T^2}{4}$	(4 - 1)25 = 75	$\frac{SS}{75}$	Var (weekly averages)

[a] The mean square for each source of variation is the corresponding sum of squares divided by the degrees of freedom.

Table B-2. ANALYSIS OF VARIANCE FOR DATA SET U-1

Source of variation	Sum of squares (SS)	Degrees of freedom (DF)	Mean Square (MS)
Among "monthly averages	0.277	24	0.0115
Among weekly averages within month	0.281	75	0.0037

Var (weekly averages within a month) ≈ 0.0037
Var (monthly averages) ≈ (0.0115 - 0.0037)/4 = 0.0020
Standard deviation of weekly averages ≈ 0.061
Standard deviation of monthly averages ≈ 0.045

The results of this type of analysis for several data sets are reported in Table B-3 for all coals, and in Table B-4 for coals with less than 1 percent sulfur. Table B-5 lists the expected values and limiting values of the variance components. The highest limiting values were obtained by using the second highest as the upper value and the second lowest as the lowest value within the week (or month) under the heading of "unit train" and also for monthly components of variation.

In previous discussions the distribution of the analyses of composite samples was assumed to be lognormal or inverted gamma rather than normal. This would suggest a transformation of the data, such as lnX, with all subsequent analyses performed on the logarithms and the results transformed back to $X = e^{\ln X} = e^{Y}$. This was done on one typical example and the difference in the results was very small. Although for this specific analysis the transformation does not seem to be justified, there will be situations in which such a transformation would be beneficial.

B.1.3 Estimation of Standard Deviation for Various Averaging Times

One of the important calculations required in Section 5 involves estimating the variability of average sulfur content based on various averaging periods of one month. Discussions of this type of computation appear in several statistical texts under the subject of nested models and components of variance. A brief description of the approach is given here.

First of all, a linear model is assumed to describe the behavior of the data, that is, the individual sample result (x) is given by the relationship;

Table B-3. ESTIMATION OF MEANS AND STANDARD DEVIATIONS

(COMPONENTS OF VARIATION) OF SULFUR CONTENT

Data Set	Mean, (weight percent)	Absolute standard deviation and relative standard deviation s (weight percent sulfur) RSD (%)							
		Unit train[a]		Weekly ( )[b]		Monthly ( )		Annually ( )	
		s	RSD	s	RSD	s( )	RSD	s ( )	RSD
C-1	2.80	0.22	7.9	-	-	0.12(20)	4.3	0.07(240)	2.5
C-2	0.61	0.026	4.4	-	-	0.015(9)	2.5	-	
C-3	0.80	0.076	9.5	-	-	0.031(13)	3.9	-	
C-4	1.38	0.030	2.9	-	-	0.32(18)	23.2	-	
C-5	0.42	0.033	8.2	-	-	0.033(10)	8.2	-	
C-8	2.60	0.13	5.0	-	-	0.08(8)	3.1	0.03(100)	1.2
U-1	0.74	0.106	14.3	0.06(7)	8.1	0.044(28)	5.9	-	
U-2	0.88	0.045	5.1	-	-	0.040(11)	4.5	-	
U-3	0.83	0.135	16.3	-	-	0.096(4)	11.6	-	
U-4	1.04	0.132	12.7	-	-	0.065(11)	6.9	-	
U-5	0.92	0.132	14.3	-	-	0.069(11)	7.4	-	
U-6	0.79	0.134	17.0	-	-	0.078(7)	9.9	-	
U-7	0.81	0.165	20.3	-	-	0.044(10)	5.4	-	
U-8	0.84	0.232	27.8	-	-	0+ (6)	0+	-	
U-9	0.80	0.164	20.0	-	-	0+ (7)	0+	-	
U-10	1.74	0.169	9.7	-	-	0.249(5)	14.3	-	
U-11	3.10	0.22	7.1	0.144(5)	4.6	0.27(20)	8.7	-	
U-12	2.25	0.157	7.0	0.166(5)	5.2	0.109(2)	4.8	-	
U-13	2.15	0.125	5.8	0.088(5)	4.1	0.129(20)	6.0	-	
Average standard deviation - s		0.09[c]		0.102		0.121[c]		0.05	
Average standard deviation RSD (% of mean)			11.4		5.5		6.9		1.9

[a] Variation among composite samples from a unit train within a week or month depending on the subsequent components of variation.

[b] Values in parentheses denote the average number of samples (measurements) per week month or year; the number is rounded to the nearest integer.

[c] These values were determined by adding the product of each s^2 by its corresponding degrees of freedom, and dividing this sum by the total degrees of freedom.

Table B-4. ESTIMATION OF MEANS AND STANDARD DEVIATIONS (COMPONENTS OF VARIATION) OF SULFUR CONTENT

(Coals with less than 1 percent sulfur)

Data Set	Mean, (weight percent)	Absolute standard deviation (s) and relative standard deviation (RSD)					
		Unit train[a]		Weekly		Monthly	
		s	RSD	s	RSD	s	RSD
U-1	0.74	0.106	14.3	0.06	8.1	0.044	5.9
U-2	0.88	0.045	5.1	-	-	0.040	4.5
U-3	0.83	0.135	16.3	-	-	0.096	11.6
U-4	1.04	0.132	14.0	-	-	0.065	6.9
U-5	0.92	0.132	14.3	-	-	0.069	7.4
U-6	0.79	0.134	17.0	-	-	0.078	9.9
U-7	0.81	0.164	20.3	-	-	0.044	5.4
U-8	0.84	0.232	27.8	-	-	0+	0+
U-9	0.80	0.164	20.0	-	-	0+	0+
C-2	0.61	0.026	4.4	-	-	0.015	2.5
C-3	0.80	0.076	9.5	-	-	0.031	3.9
C-5	0.42	0.033	8.2	-	-	0.033	8.2
Average standard deviation		0.109[b]		0.06		0.056	
Average relative standard deviation (% of the mean)			14.3[c]		8.1		5.9

[a] Variation among composite samples from a unit train within week or month depending on the subsequent component of variation.

[b] These values were determined by adding the product of each s^2 by its corresponding degrees of freedom, and dividing this sum by the total degrees of freedom.

[c] These averages are ordinary averages, not weighted by the degrees of freedom or the number of samples analyzed. The data sets represent coals having varying degrees of mixing in the mining operation and loading on cars, a wide range of tonnages of coal sampled, and varying coal preparation/processing/cleaning procedures. In particular, data sets C-2 and C-3 are washed coals. Hence, the average value of the RSD does not correspond to that of a given coal but is a mid-range value.

Table B-5 EXPECTED VALUES AND LIMITING VALUES FOR ABSOLUTE AND RELATIVE STANDARD DEVIATION FOR EACH COMPONENT OF VARIANCE

Component of variation	Lower limit		Expected value		Upper limit	
	s	RSD	s[b]	RSD[c]	s	RSD
Among unit trains[a]	0.033	5.1%	0.109	14.3%	0.169	20.3%
Among weeks within months	-[d]	-	0.06	8.1%	-	-
Among months	0.031	3.9%	0.056	5.9%	0.078	9.9%

[a] Variation among composite samples from a unit train within week or month depending on the subsequent component of variation.

[b] These values were determined by adding the product of each s^2 by its corresponding degrees of freedom, and dividing this sum by the total degrees of freedom.

[c] These averages are ordinary averages, not weighted by the degrees of freedom or the number of samples analyzed. The data sets represent coals having varying degrees of mixing in the mining operation and loading on cars, a wide range of tonnages of coal sampled, and varying coal preparation/processing/cleaning procedures. In particular, data sets C-2 and C-3 are washed coals. Hence, the average value of the RSD does not correspond to that of a given coal but is a mid-range value.

[d] Insufficient data

x = Long-term average (e.g., an average for a year (s))
+ monthly effect (deviation of monthly average from the long-term average), α_m
+ within month effect (deviation of unit train result from the monthly average), β_{wm}

$$x = \mu + \alpha_m + \beta_{wm}, \qquad \text{Equation B-1}$$

or

x = Long-term average
+ monthly effect (deviation of monthly average from the long-term average), α_m
+ weekly effect (deviation of weekly averages from their monthly average), β_{wm}
+ within weeks effect (deviation of the individual sample result from the weekly average), δ_{ww}

$$x = \mu + \alpha_m + \beta_{wm} + \delta_{ww} \qquad \text{Equation B-2}$$

It is assumed that α_m, β_{wm}, δ_{ww} are all normally and independently distributed with zero means and variances σ_α^2, σ_β^2, and σ_δ^2, respectively. In theory, another effect can be added to equation B-2, that is, the effect of analytical variation due to imprecision of the measurement technique. However, this effect is included in the component of variation, δ_{ww}, and there is no means of separating this effect unless these were split samples and sulfur measurements were performed on each sample. The differences between these measurements would be a measure of the precision of the sulfur measurement; in fact, the square of this difference divided by 2 would be an estimate of the variance component due to the measurement itself. The component of variance, σ_δ^2, includes two components, one due to the sulfur variability, σ_δ^2, and one due to the analytical measurement, σ_A^2, that is,

$$\sigma_\delta^2 = \sigma_S^2 + \sigma_A^2.$$

Since there is no convenient means of separating these two components for these data, they are treated as one component throughout this report. For example, if the standard deviation of sulfur variation were 0.12 and that for the analytical technique were 0.05, then the combined variance would be $\sigma_\delta^2 = 0.0144 + 0.0025 = 0.0169$ or $\sigma_\delta = 0.13$. Hence, if the analytical error is small in comparison to the sulfur variability, the effect of treating the two components in combination is small.

Subject to the above assumptions the variance of the result, x, is given by the sum of the variance of each component. Hence the variance of x is given by

$$\mathrm{Var}(x) = \mathrm{Var}\ (\alpha_m) + \mathrm{Var}\ (\beta_{wm}) \qquad \text{Equation B-3}$$

or by

$$\mathrm{Var}(x) = \mathrm{Var}(\alpha_m) + \mathrm{Var}(\beta_{wm}) + \mathrm{Var}(\delta_{ww}) \qquad \text{Equation B-4}$$

corresponding to equations B-1 and B-2, respectively. The $\mathrm{Var}(\mu) = 0$ as μ is considered a constant, a long-term average weight percent sulfur.

Now suppose that eight measurements are obtained per month and their averages computed, the expected variance of the average is given by

$$\mathrm{Var}(\bar{x}, \text{monthly}) = \sigma^2_\alpha + \frac{\sigma^2_\beta}{8}$$ Equation B-5

where σ^2_α and σ^2_β are the variances of α_m and β_{wm}, respectively, and equation B-1 is the assumed model. That is, the component of variability within months is divided by the sample size (8 in this case) and the monthly component is added to this value to obtain the total variance. The standard deviation is obtained by taking the square root of the variance. In general, the variance for the average of n measurements per month is given by,

$$\mathrm{Var}(\bar{x}, \text{monthly}) = \sigma^2_\alpha + \frac{\sigma^2_\beta}{n} ,$$ Equation B-5

and for 3 months (quarterly),

$$\mathrm{Var}(\bar{x}, \text{quarterly}) = \frac{\sigma^2_\alpha}{3} + \frac{\sigma^2_\beta}{3n} .$$ Equation B-6

The estimates of these variances are obtained by substituting the corresponding estimates obtained from Table B-4 (coals with less than 1 percent sulfur).

An example calculation is given to illustrate the use of the above approach to calculate the entries in Table 5-1, under the column headed by w = 4, n = 16. From Table B-4, the following RSD's are obtained:

Among months, RSD = 5.9%.

Among weeks within months, RSD = 8.1%
(data for one set only).

Among unit trains within weeks (or months), RSD = 14.3%.

From these data the following RSD's are obtained assuming 4 unit trains per week and 16 unit trains per month

$$\text{RSD(unit trains} = (14.3^2 + 5.9^2)^{1/2} = 15.5\%.$$

$$\text{RSD(weeks)} = (5.9^2 + 8.1^2 + \frac{14.3^2}{4})^{1/2} = 12.3\%.$$

$$\text{RSD(months)} = (5.9^2 + \frac{14.3^2}{16})^{1/2} = 6.9\%.$$

$$\text{RSD(quarterly)} = (\frac{5.9^2}{3} + \frac{14.3^2}{48})^{1/2} = 4.0\%.$$

$$\text{RSD(semi-annually)} = (\frac{5.9^2}{6} + \frac{14.3^2}{96})^{1/2} = 2.8\%.$$

$$\text{RSD(annually)} = (\frac{5.9^2}{12} + \frac{14.3^2}{192})^{1/2} = 2.0\%$$

B.1.4 Correlation Analysis

It is also of interest to determine whether the weight percent sulfur (S) and Btu content (B) are correlated. To determine this correlation, S and B values are cross-tabulated, as shown in Table B-6; a scatter diagram of the same data set is given in Figure B-1. The simple correlation is given by

$$r = \frac{\Sigma(B_i - \bar{B})\ (S_i - \bar{S})}{\sqrt{\Sigma(B_i - \bar{B})^2 \times \Sigma(S_i - \bar{S})^2}}$$

and for these data, the value for r is 0.18.

Table B-6. CROSS-TABULATION OF SULFUR AND BTU CONTENTS

Btu/lb	Weight percent sulfur (S)						
	< 0.65	0.66-0.69	0.70-0.73	0.74-0.77	0.78-0.81	0.82-0.85	> 0.85
< 12,100	5	6	8	4	0	1	2
12,100-12,200	5	4	4	4	1	0	1
12,200-12,300	1	4	4	2	0	0	4
12,300-12,400	3	4	5	0	1	1	1
12,400-12,500	3	1	5	2	3	1	3
12,500-12,600	5	5	2	0	1	0	6
12,600-12,700	2	3	5	1	2	2	3
12,700-12,800	4	1	1	1	3	0	4
12,800-12,900	6	3	4	2	1	4	6
> 12,900	2	4	3	3	1	3	3

The correlation coefficient r is given by

$$r = \frac{\{\Sigma(B - \bar{B})(S - \bar{S})\}/(N - 1)}{\left[\frac{\Sigma(B - \bar{B})^2}{N - 1} \times \frac{\Sigma(S - \bar{S})^2}{N - 1}\right]^{\frac{1}{2}}} = \frac{4.603}{\left[(303.1) \times (0.0839)\right]^{\frac{1}{2}}} = 0.181^{a}$$

[a] Referring to a standard table[8] for confidence intervals for the correlation coefficient, it is concluded with 95 percent confidence that the correlation is slightly positive, that is, between about 0.04 and 0.35.

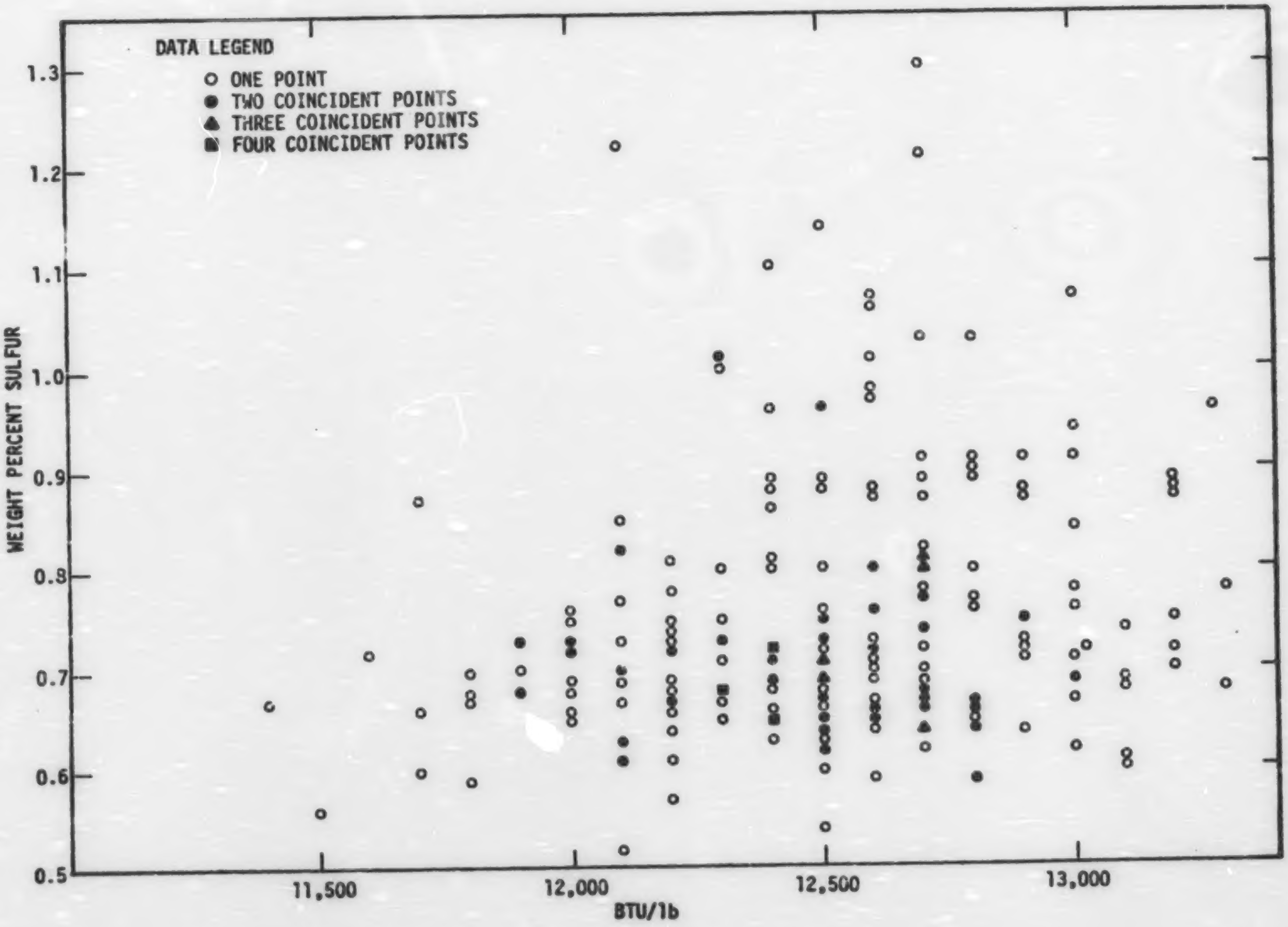

Figure B-1. Scatter diagram of a sample of data set U-1.

B.1.5 Estimated Variance of lb SO_2/MM Btu

The estimated standard deviation (or variance) of a ratio can be obtained very simply using the relative standard deviations and the relationship,

$$\text{Relative Variance } (kS/B) \approx \text{Rel. Var}(S) + \text{Rel. Var}(B),$$

provided S and B are statistically independent.[6] A cross-plot of S and B for several data sets indicates that independence is a reasonable assumption. Now assume that the relative standard deviation of S is about 15 percent and that for B it is about 5 percent (a conservative, large value). The relative variances are the squares of these values; it is readily seen that the effect of the variation in B is small in terms of the relative variance in the ratio kS/B versus the relative variance in S above. For the specific example,

$$\text{RSD of } S = 15$$

$$\text{RSD of } kS/B = (15^2 + 5^2)^{1/2} = 15.81,$$

a 5.4 percent increase in the relative standard deviation. In this report it is assumed that the RSD of the ratio kS/B is equal to 1.05 times that for S. This is a matter of convenience. One must be careful not to make this assumption for the absolute standard deviation of the ratio (kS/B), which would increase by a factor of about k times that for weight percent sulfur (S).

B.1.6 Relationship Between the Average, $\bar{x}$ and Standard Deviation, s

In order to study the relationship between s and $\bar{x}$, a plot was made for s versus $\bar{x}$, as shown in Figure B-2. This figure clearly indicates the dependence of s on $\bar{x}$, which is best approximated by a linear relationship, particularly for coals with sulfur concentrations less than 1.5 weight percent. The utility of the lognormal transformation is based on this indicated dependence of s on $\bar{x}$.

B.2 DETAILED ANALYSIS OF DATA SET U-1 (% SULFUR, DRY BASIS)

A more detailed analysis was conducted of the weight percent sulfur, Data Set U-1, to estimate the frequency distribution of sulfur in the coal as-received at a specific plant.

To simplify the analysis the 103 values for weekly weighted averages, and the weekly ranges (largest value less the smallest value) were tabulated as shown in Appendix A, Data Set U-1. These two frequency distributions were then plotted on lognormal probability graph paper; experience with several data sets has shown that the lognormal distribution is a reasonable empirical approximation of the frequency distribution of sulfur. The results of the Navajo Report[1] are also used to obtain empirical frequency distribution. There is no reason that the range should be lognormally distributed but the plotted frequency distribution still provides a convenient summary of the data. Figure B-3 gives the graphical presentation.

The data were subdivided by approximate periods of one-half year each as an aid in determining any major long-term trends. The next step in the analysis was to compute the means, $\bar{x}$, and standard deviations, s, of the weekly weighted averages for each 6-month period (approximate) and for the 2-year period. These results are given in Appendix A, Data Set U-1, which also gives the mean ranges for the data.

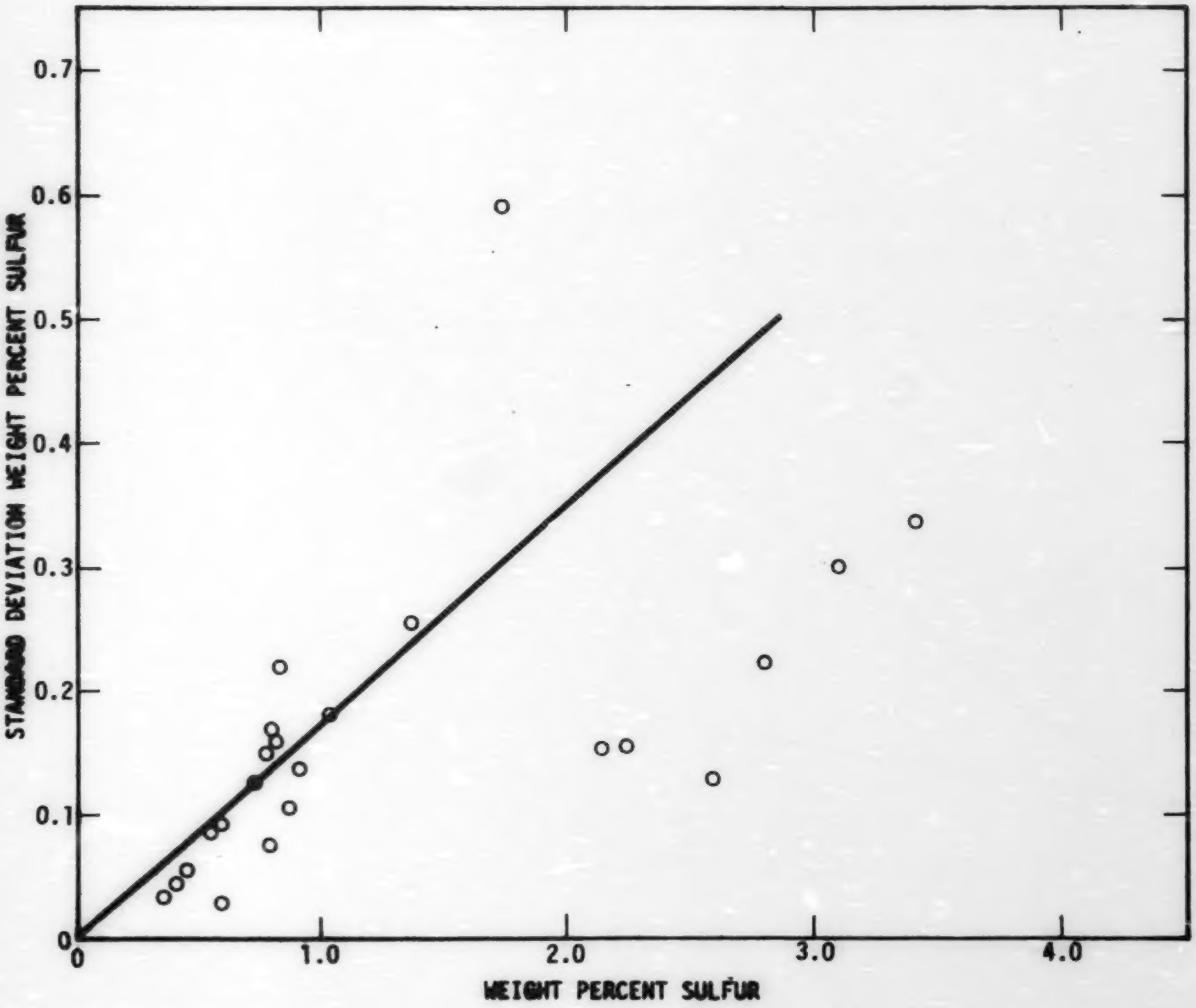

Figure B-2. Relationship of standard deviation to mean sulfur content.

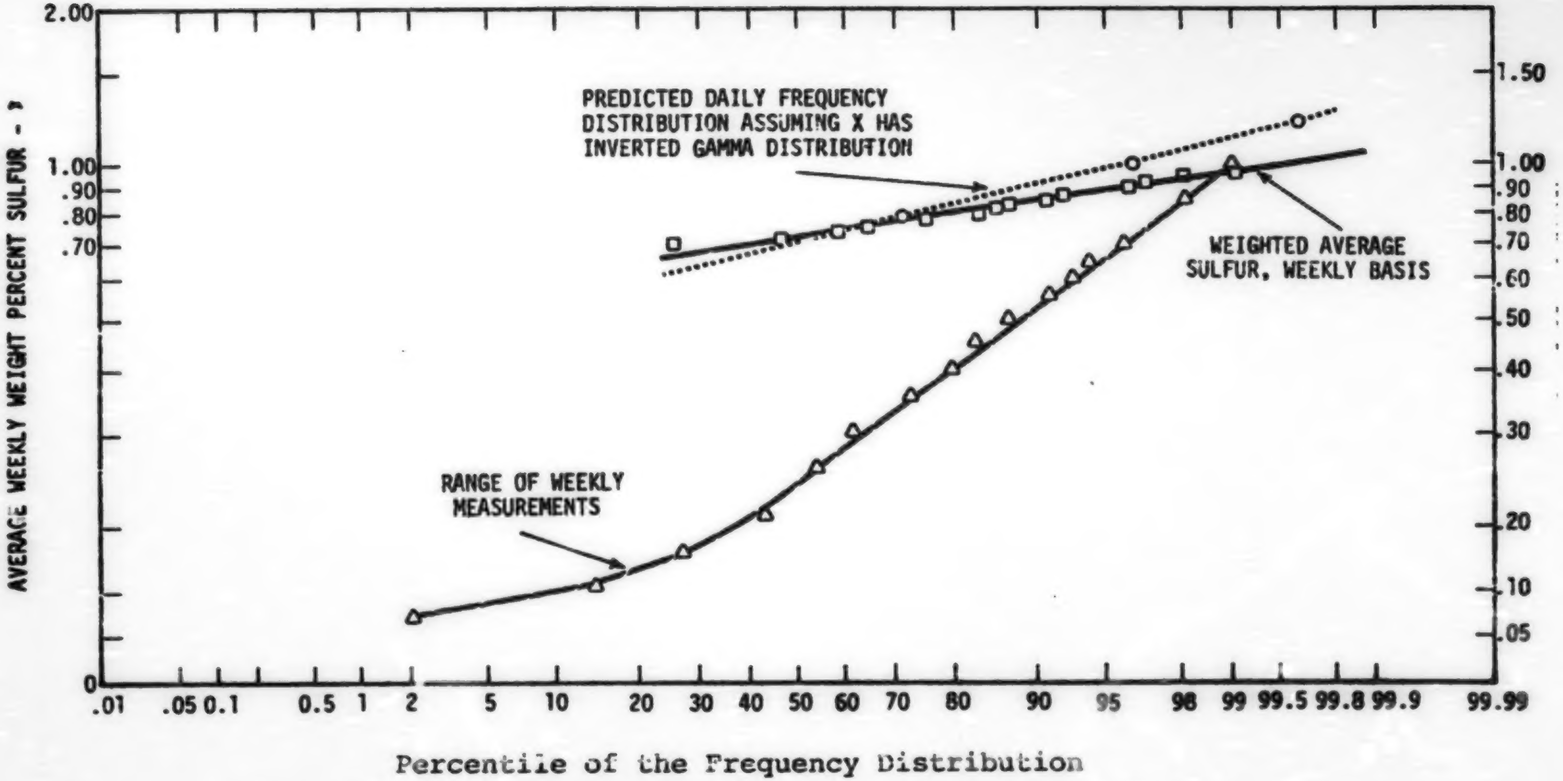

Figure B-3. Frequency distribution for Data Set U-1.

The overall mean sulfur content is $\bar{\bar{x}} = 0.742$ and the standard deviation of the weekly weighted averages is $s = 0.0732$. A separate analysis was conducted for the ln x (natural log of sulfur content, weight percent); the geometric mean and standard deviation are 0.762 and 0.0866. The overall mean weekly range, $\bar{R}$, is 0.287. From this value an estimate of the standard deviation, $\hat{\sigma}$, within weeks is $\hat{\sigma} = \bar{R}/d_2 = 0.287/\ 2.701 = 0.106$, where d_2 is the factor that when divided into the mean range yields an estimate of the standard deviation, $\hat{\sigma}$ (see reference 8).

With these results it is easy to estimate (again by approximate methods) the standard deviation of individual sulfur measurements on a daily basis. The total variance of a daily measurement is approximately

$$\begin{aligned}\hat{\sigma}^2 \text{ (Total)} &= \hat{\sigma}^2 \text{ (within weeks)} + s^2 \text{ (among weeks)} \\ &= (0.106)^2 + (0.0732)^2 \\ &= 0.0166 \\ \hat{\sigma} \text{ (Total)} &= 0.13\end{aligned}$$

Another line is then shown on the logarithmic probability paper which has the same median value (50th percentile) but with a slope consistent with the variability of the daily measurements, $\hat{\sigma}$ (Total). This is done by plotting, $\bar{\bar{x}} + \hat{\sigma}$ at the 84th percentile and $\bar{\bar{x}} + 2\hat{\sigma}$ at the 97.5th percentile. This line provides a reasonable approximation of the distribution that would have been obtained if all of the data were plotted as was done for the weekly average values. (This would have required considerable additional work.)

The next consideration in the analysis was utilizing the background information given in the Navajo report.[1] Therein it is explained that the weight percent sulfur of the coal to be burned, x, has approximately an inverted gamma distribution, the form of which is given below for reference only. It is not used directly in subsequent calculations,

$$f(w) = (\beta\lambda)^{\beta}\left(\frac{1}{x}\right)^{\beta+1} e^{-\beta\lambda/x}/\Gamma(\beta), \quad x > 0, \ \beta,\lambda > 0.$$

The results of using this distribution will be compared to those using the lognormal distribution.

In order to apply the above distribution, it is necessary first to estimate the parameters β and λ or β and $\beta\lambda = \alpha$, by using the following relationships given in Reference 1.

$$\begin{cases} \bar{\bar{x}} = \dfrac{\hat{\alpha}}{\hat{\beta} - 1} \\ \hat{\sigma}^2(\text{Total}) = \dfrac{\hat{\alpha}^2}{(\hat{\beta} - 1)^2(\hat{\beta} - 2)} \end{cases}$$

where $\bar{\bar{x}}$ = the overall mean = 0.742 and $\hat{\sigma}$(Total) = 0.129, $\hat{\sigma}^2$(Total) = 0.0166. Substituting in the first equation above,

$$0.742 = \frac{\hat{\alpha}}{\hat{\beta} - 1}$$

and thus

$$0.0166 = (0.5506)\frac{1}{\hat{\beta} - 2}$$

or $\hat{\beta} - 2 = 33$, $\hat{\beta} = 35$.

Hence, $\hat{\alpha} = 0.742(\beta - 1) = 25.23$.

Incomplete gamma function tables are not conveniently available, whereas χ^2 tables (for special cases of the parameter β) are readily available.[8] Hence, the probability that x, the weighted percent sulfur for a daily sample, is greater than some specified value can be obtained directly by interpolation in a table for χ^2 as shown in the example computation given below. The probability is estimated by

$$\Pr\{x > x_o\} = \Pr\{\chi^2 < \frac{2\alpha}{x_o} | f = 2\beta\},$$

where the example data

$2\hat{\alpha} = 50.46$

$2\beta = 70 = \nu =$ degrees of freedom used with the χ^2 table ($\nu = f$ in Hald's table[8]),

For example, if $x_o = 1$,

$$\Pr\{\chi^2 < \frac{50.46}{1} = 50.46 | \nu = f = 70 \text{ degrees of freedom}\}$$
$$= 2.5 + \left(\frac{50.46 - 48.8}{2.9}\right)2.5 = 3.93 \text{ or about } 4\%.$$

If $x_o = 1.2$,

$$\Pr\{\chi^2 < \frac{50.46}{1.2} = 42.05 | \nu = 70\} = 0.38\%.$$

If $x_o = 0.8$,

$$\Pr\{\chi^2 < \frac{50.46}{0.8} = 63.08 | \nu = 70\} = 29.35\%.$$

These values may be plotted on log probability paper. Theoretically they will not fall on a straight line, but the approximation will be good. Note that the points almost coincide with the straight line drawn on the graph based on the lognormal assumption. Thus it is very difficult to discriminate between two such empirical approximations. The lognormal frequency distribution is more convenient, and the results are essentially identical to those obtained with the inverted gamma distribution.

B.3 AVERAGES FOR FEWER THAN FIVE MEASUREMENTS ASSUMING INVERTED GAMMA DISTRIBUTION

Section 5.3.3 refers to the fitting of the inverted gamma frequency distribution to the case where fewer than five measurements are averaged. A simple technique for fitting an inverted gamma to a data set is given herein. The two parameters, α and β, of the inverted gamma are estimated by the relationships

$$\bar{x} = \frac{\hat{\alpha}}{\hat{\beta} - 1}$$

$$s^2 = \hat{\sigma}^2/(\hat{\beta} - 1)^2 \ (\hat{\beta} - 2)$$

Combining the two equations yields a simple system of equations, i.e.,

$$\bar{x} = \frac{\hat{\alpha}}{\hat{\beta} - 1}$$

$$(\mathrm{RSD})^2 = \frac{s^2}{\bar{x}^2} = \frac{1}{\hat{\beta} - 2} \ .$$

The value of m is determined by iteration starting with the value estimated by the lognormal assumption. Thus for a given RSD, $\hat{\beta}$ can be readily determined and substituted in the first equation to solve for $\hat{\alpha}$. For example, for RSD = 0.204, and $\bar{x}$ = 0.858 (first iteration) we obtain

$$0.0416 = \frac{1}{\hat{\beta} - 2}$$

or

$$\beta - 2 = 24.0$$
$$\hat{\beta} = 26.0$$
$$\hat{\alpha} = 21.48.$$

From further results given in Appendix B.2, the probability that x is greater than the emission standard, 1.2 lb SO_2/MM Btu, is given by the following relationship

$$P(x > 1.2) \approx \Pr\{\chi^2 < \frac{2\hat{\alpha}}{1.2} \mid f = 2\hat{\beta}\}$$

$$= \Pr\{\chi^2 < 35.8 \mid f = 52\},$$

where χ^2 is the chi-square variable for which the distribution is tabulated in reference 8 and f is the associated number of degrees of freedom. Determination of the probability requires interpolation within the χ^2 table,

	Probability			
f	0.01	0.025	0.05	0.10
52.0	31.2	34.0	36.4	39.4

The probability is estimated to be 0.044 for this mean. A larger mean can be selected and the new probability calculated. Repeat this process until the mean is consistent with a probability of 0.05. Hence if m = 0.87, $\hat{\beta} = 26.0$ as above but $\hat{\alpha} = 21.75$.

$$\Pr\{\chi^2 < 36.2 \mid f = 52\} = 0.0480$$

If m = 0.88

$$\Pr\{\chi^2 < 36.7 \mid f = 52\} = 0.0528.$$

Therefore m = 0.875 is the mean required for 95 percent compliance with 1.2 lb SO_2/MM Btu. This mean is slightly larger than the mean estimated using the lognormal assumption. This same close relationship between the two assumptions was obtained for all of the example computations; hence, the lognormal assumption is both adequate and somewhat conservative.

B.4 APPLICATION OF QUALITY CONTROL TO SULFUR VARIABILITY

Suppose that it is desired to control the long-term average sulfur content at some specified level, say 0.987 lb SO_2/MM Btu. Furthermore, it is desired to control the variation about this average within limits that are considered reasonable for "inherent" variability of sulfur content of coals. These requirements immediately suggest a reasonably simple application of quality control techniques to, for example, monthly averages of sulfur content.

On the basis of data in Section 5, the estimated relative standard deviation of monthly averages of weight percent sulfur based on 16 unit trains was determined to be about 6.9 percent. Based on the data of Section B.1.5, Appendix B, the relative standard deviation of monthly averages of lb SO_2/MM Btu would be slightly higher, say 6.9 (1.05) ≈ 7.2 percent of the mean value. Hence, if it is desired not to exceed a limit of 1.2 lb SO_2/MM Btu except for a very small portion of the monthly averages determined by an upper (3σ) control limit, then the required mean sulfur content would be

$$\frac{1.2 - m}{0.072\ m} = 3 \text{ (corresponding to } 3\sigma \text{ limit)}$$

or

$$m = 0.987 \text{ lb } SO_2\text{/MM Btu}$$

The mean (or center) line on the control chart would be 0.987, the upper 2σ limit (warning limit) would be

$$0.987 + 2(0.072)(0.987) = 1.13 \text{ lb } SO_2\text{/MM Btu}$$

and the upper control limit would be

$$0.987 + 3(0.072)(0.987) = 1.20 \text{ lb } SO_2\text{/MM Btu}$$

as prescribed in the above computation and shown in Figure B-4. The lower warning and control limits are similarly obtained by replacing the plus (+) sign by a minus (-) sign in the above equations. If the mean sulfur content remains at 0.987 lb SO_2/MM Btu, and the RSD remains at or below 0.071 (0.072 x 0.987), then the chance of a monthly average exceeding 1.2 is very small, less than 2 chances in 1000 if the average can be assumed to be normally distributed. This is a reasonable assumption for averages of several unit train measurements, as previously indicated. It is not a good assumption for individual unit train analyses or for averages of less than five composite samples.

Both warning limits are included on the chart corresponding to the expected variation determined by analyses of several data sets, Table 5-1. The upper control should be used as a signal to take immediate corrective action in that the sulfur content is considerably high and that, for example, a coal of lower sulfur content should be used or mixed in appropriate proportion with the current coal to yield a long-term maximum of 1.2 lb SO_2/MM Btu. The upper warning limit may be considered a signal to take action if two consecutive values fall between the upper warning and control limits. Corrective action may also be taken if a series of seven consecutive values lie above the mean value line 0.987, or if there is a series of seven or more consecutively increasing values, even if some of the values are below the mean line at 0.987.

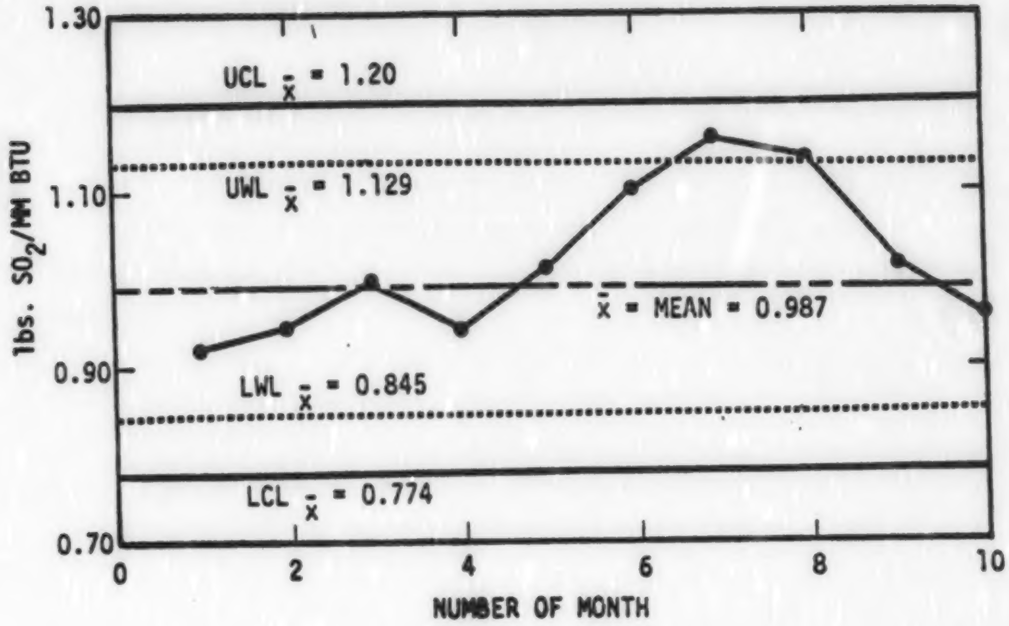

Figure B-4. Application of quality control chart to control sulfur variability.

The lower warning and control limits do not require corrective action because they indicate a desirable condition leading to lower emissions. These limits are entered on the chart to indicate a possible improvement in coal quality. From the standpoint of the supplier, it would be beneficial to know that a potential improvement in quality has resulted from, say, a change in mining operation.

The potential benefits of this approach are that (1) compliance with the standard of 1.2 lb SO_2/MM Btu is achieved for a desired percentage of the averages through careful monitoring of the sulfur content and (2) this interpretation of the NSPS permits use of the largest possible quantity of coal reserve while operating in compliance. The disadvantage is that for periods of 1 month the sulfur content of the coal may be outside the control limit before action is taken. In practice, a within-month check can be maintained by use of a moving average, as described in the following paragraphs, to allow more rapid action if the coal is tending toward the high-sulfur range.

The quality control chart can also be used for moving averages and ranges of four consecutive weekly averages or of individual analyses. Data set U-1 is analyzed to illustrate how the quality control limits are derived and used to "monitor" the sulfur content of coal. Table B-7 gives the weekly average weight percent sulfur, the 4-week moving average, and range. The quality control limits are then derived from the average of the moving average, $\bar{\bar{x}} = 0.797$ percent sulfur by weight, and of the moving range, $\bar{R} = 0.141$. The formulas given at the bottom of the table require constants as given in Grant and Leavenworth.[10] The moving average and range charts provide a moving "monthly" average and a range of the weekly values during the month, assuming 4 weeks data are averaged. These data can aid in detecting trends in the mean or variation of sulfur content. The quality control chart for this example is given in Figure B-5. There are potential benefits in plotting simultaneously a chart for the averages of the unit trains as received during one week and the monthly averages. The crossover points for these two charts are indicative of trends that may warn the company of a trend toward undesired values of sulfur content.

Table B-7. DATA AND COMPUTATION OF CONTROL CHART LIMITS FOR MOVING AVERAGE CHARTS FOR SULFUR CONTENT

(values in weight percent sulfur)

Number of week	Weekly average	Four-week moving average	Four-week moving range
1	0.72		
2	0.76		
3	0.73		
4	0.83	0.760	0.11
5	0.85	0.793	0.12
6	0.71	0.780	0.14
7	0.78	0.793	0.14
8	0.74	0.770	0.14
9	0.72	0.738	0.06
10	0.70	0.735	0.08
11	0.78	0.735	0.08
12	0.84	0.760	0.14
13	0.78	0.775	0.14
14	0.88	0.820	0.10
15	0.94	0.860	0.16
16	0.99	0.898	0.21
17	0.78	0.898	0.21
18	0.78	0.873	0.21
19	0.71	0.815	0.28
20	0.70	0.743	0.08
Total		13.543	2.40
Average		$\bar{\bar{x}} = 0.797$	$\bar{R} = 0.141$

$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 0.80 + 0.73(0.14) = 0.90$

$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 0.80 - 0.73(0.14) = 0.70$

$UCL_R = D_2\bar{R} = 2.28(0.14) \simeq 0.32$

$LCL_R = D_3\bar{R} = 0(0.14) = 0$

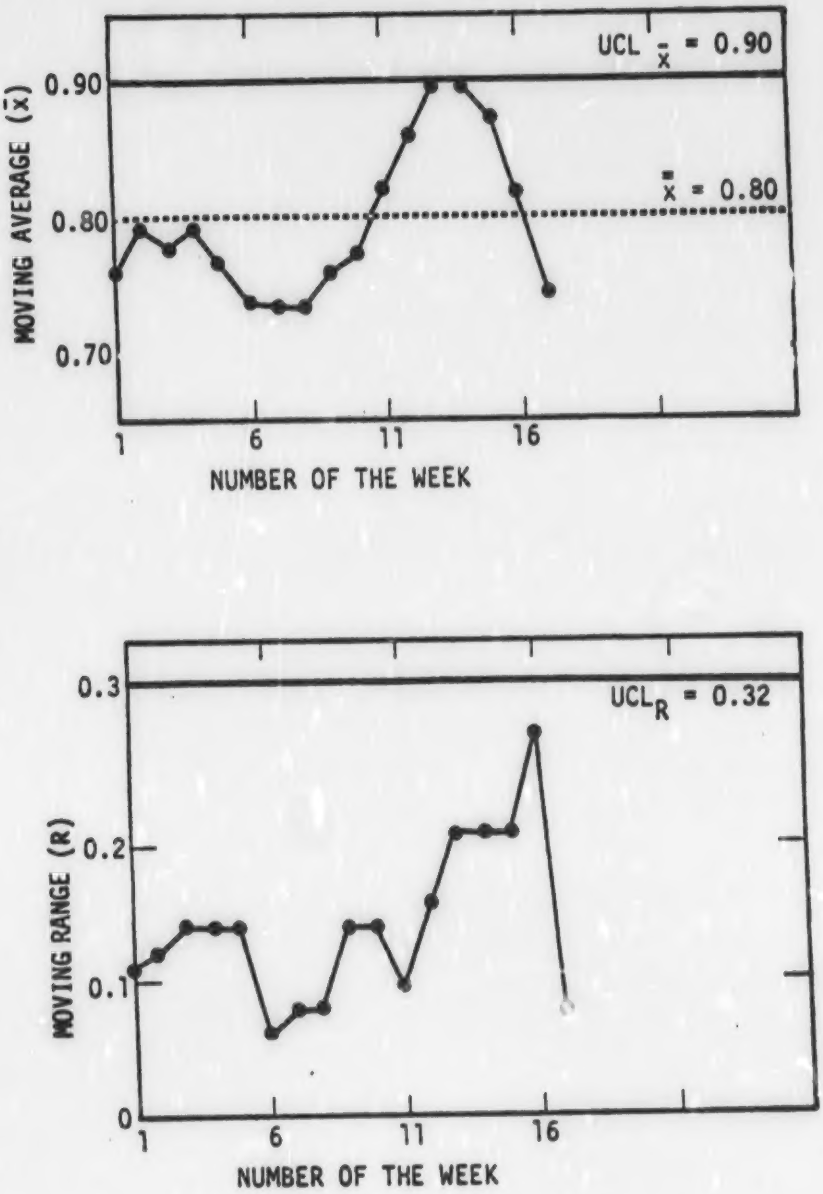

Figure B-5. Moving average and moving range quality control charts

APPENDIX C

SULFUR VARIABILITY WITH LOT SIZE

C.1 INTRODUCTION

Performance of power plants in regard to SO_2 emissions is generally governed by the New Source Performance Standards (NSPS) applicable to new plants commissioned after 1975 or by a State Implementation Plan (SIP) applicable to existing plants. These standards normally limit SO_2 emissions to a value expressed in pounds of SO_2 per million Btu (lb/MM Btu), a value never to be exceeded. The test procedure prescribed for ascertaining whether a plant is in compliance implies, however, that the value for lb/MM Btu actually refers to a 3-hour average.

The standard, interpreted in common terms of 3-hour averages, has different impacts on different sizes of power plants. A typical 500-MW plant consumes about 600 tons of coal in a 3-hour period, whereas the smallest plant that comes under the purview of NSPS, a 25-MW plant (or 250×10^6 Btu/hr), consumes only 33 tons of coal in 3 hours. Thus, a 500-MW plant consumes a larger tonnage of coal with which to achieve the 3-hour average emission rate. Compliance is more difficult for the smaller plant, which has only 33 tons of coal over which it can average to meet the standard. As a corollary, the small plant also will have a comparatively more difficult task in meeting the standard during nonpeak hours. This is so because during peak hours the plant consumes considerably greater quantities of coal than it does during nonpeak hours.

The compliance problems of small plants are further compounded by yet another factor, the greater variability of sulfur content of composite samples from smaller lot sizes. If truly representative samples are collected from 600-ton lot sizes, the variance of the sulfur measurements would be smaller than that in 33-ton lot sizes. This situation is depicted graphically in Figure C-1. Theoretically, the above statement derives support from the Central Limit Theorem, which would assert that the sulfur content of samples from 33-ton lot sizes (as well as the sums) would have greater variance than that for 100-ton and 600-ton lot sizes. The larger lot sizes have a less variance because of the blending involved in collection of representative samples from progressively larger lot sizes. Thus, the smaller plants, because of the greater spread of their sulfur content measurement, would have to start with smaller nominal sulfur content if a certain standard is not to be exceeded. Alternatively, if two plants burn coals with the same nominal sulfur content from the same mine the smaller plant would produce a larger number of SO_2 emission violations than would the larger plant.

C.2 STATISTICS OF NORMAL DISTRIBUTION

Although lognormal and inverted gamma distributions have been proposed and used in connection with SO_2 emissions, for purposes of this discussion a normal distribution of emissions is assumed. If one of the skewed distributions were used, the discussion that follows would have still greater implications in regard to the number of violations.

Statistics of normal distribution are shown in Figure C-2, where the figures with arrows to the right indicate areas under the curve up to a given point and figures with arrows to the left indicate areas beyond that point. Typically, at $\mu + 3\sigma$ (where μ = mean and σ = standard deviation)

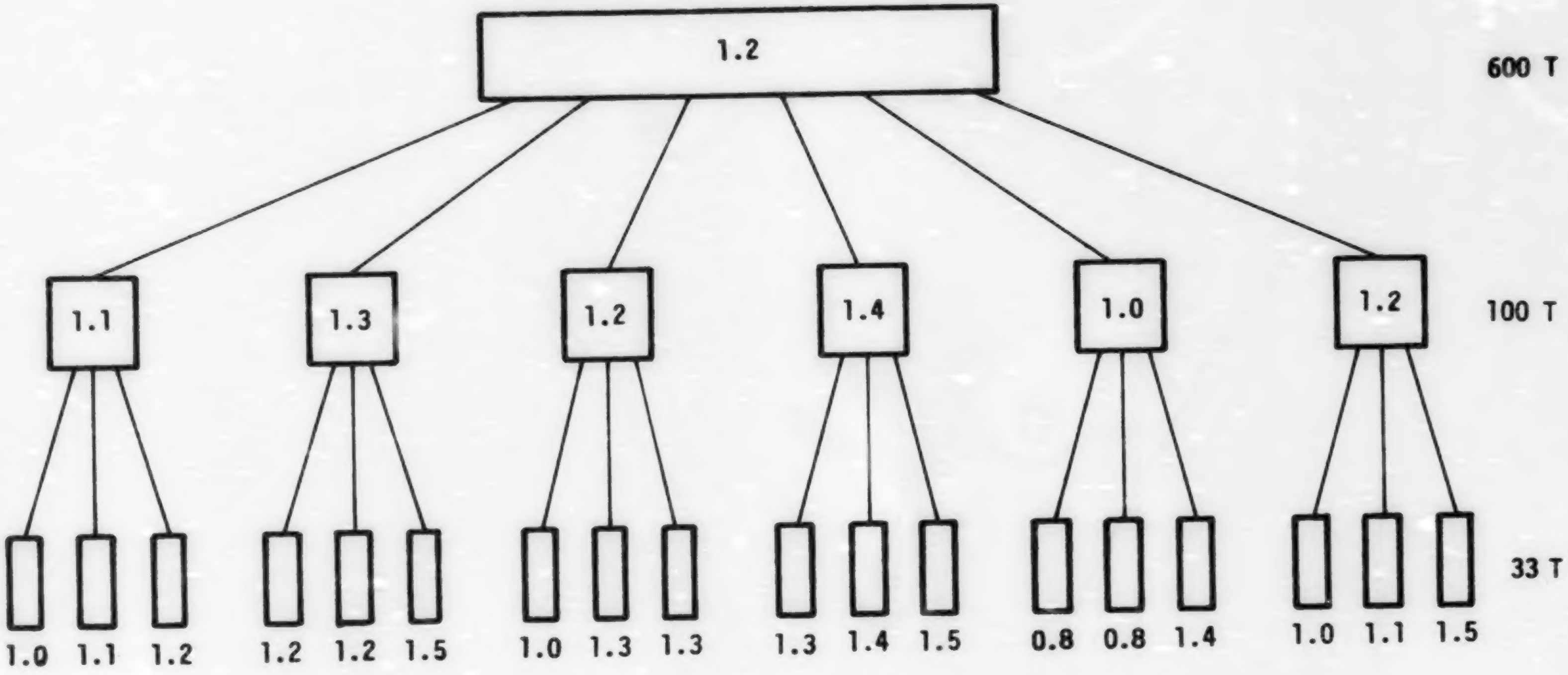
1.2
600 T
1.1
1.3
1.2
1.4
1.0
1.2
100 T
33 T
1.0
1.1
1.2
1.2
1.2
1.5
1.0
1.3
1.3
1.3
1.4
1.5
0.8
0.8
1.4
1.0
1.1
1.5

Figure C-1. Hypothetical example illustrating lot size variability.

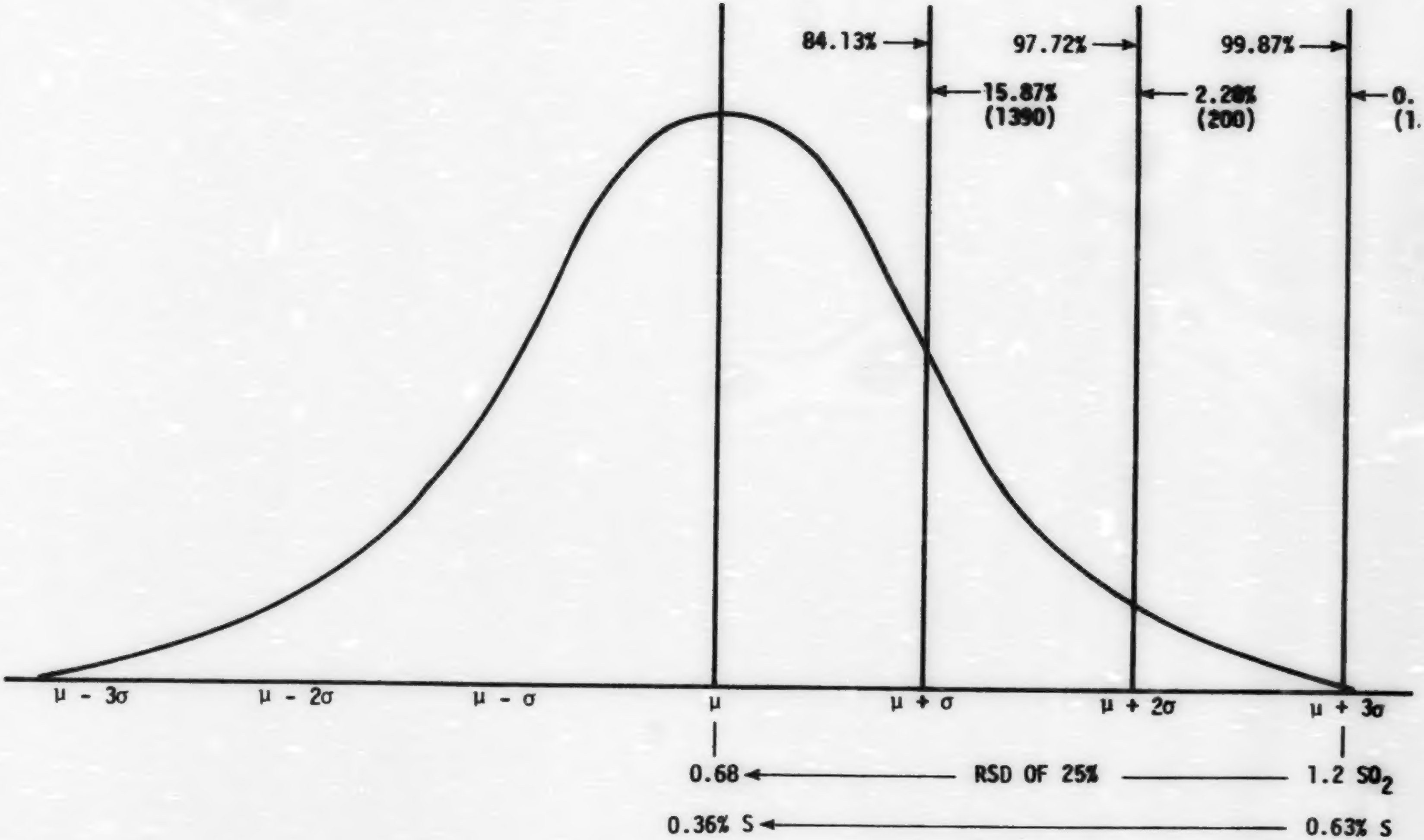

Figure C-2. Statistics for normal distribution.

the area up to that point is 99.87 percent and the area beyond the point is only 0.13 percent. Theoretically, the normal distribution is asymptotic to the x-axis, but for all practical purposes we can neglect the curve beyond $\mu + 3\sigma$. The resulting error is 0.13 percent. Although this error appears to be insignificantly small, it can represent a significant number of violations per year. If for example, a normal curve represents the distribution of a year's 3-hour overlapping averages (8760 averages/year), then about 12 of these averages would exceed the $\mu + 3\sigma$ limit (see values in parentheses on the figure).

If the emission standard is 1.2 lb SO_2/MM Btu and the coal is known to have an RSD of say 25 percent for lot sizes corresponding to a 3-hour coal consumption, then the mean value of lb SO_2/MM Btu would have to be a much smaller value, 0.68, requiring an extremely low-sulfur coal. Even then, there may be about 12 violations per year. In an attempt to reduce the number of violations, the emission standard of 1.2 lb SO_2/MM Btu may be set equal to $\mu + 3\sigma$, resulting in about two violations per year. If the standard is set equal to $\mu + 4\sigma$, then there will be about 0.28 violations per year; though negligibly small, there is the finite probability that violation is likely to occur once every 4 years. If the standard were set equal to $\mu + 4\sigma$, however, we would be faced with the concomitant problem of using coal that on an average would emit only 0.6 lb SO_2/MM Btu, requiring sulfur content of 0.32 percent with a heating value of 10,000 Btu/lb. As cited previously, if the lognormal distribution were used, the expected number of violations would be larger than the values indicated above.

C.3 IMPLICATIONS OF VARIABILITY

With regard to the sulfur variability of coal, if a certain standard is not to be exceeded except for a finite number of times, we seek a coal for which the mean sulfur content is compatible with the emission standard and the RSD of sulfur content. Larger RSD's imply that the means must be decreased. Thus, for RSD's of 10 and 25 percent with an emission standard of 1.2 lb SO_2/MM Btu set equal to $\mu + 3\sigma$, the required mean sulfur contents in lb SO_2/MM Btu would be:

$\mu_1 + 3 \times 0.10 \times \mu_1 = 1.2$ or $\mu_1 = 0.92$.

$\mu_2 + 3 \quad 0.25 \times \mu_2 = 1.2$ or $\mu_2 = 0.69$.

C.4 VARIABILITY WITH LOT SIZE

As indicated earlier, the sulfur contents of composite samples from larger lot sizes shows smaller variance of RSD. This is also indicated clearly in Figure 5-1 and Table 5-1 of this report. Quantification of the relationship between the RSD and lot sizes is, however, considerably more difficult than Figure 5-1 suggests. RSD's of different coal supplies are highly variable. Furthermore, one can assume that the RSD's of clean coals are much smaller than those of the raw coals. Although, theoretically, analyses of sulfur content of composite samples from larger lots should show smaller RSD's, analyses of actual coal data often show departures from this relationship, probably caused by local aberrations in the data. The overall trend of smaller variance with larger lot sizes may be established, but in a local sense the relationship may not be verified. Several data sets are discussed below in this context.

C.5 IDENTIFICATION OF DATA

C.5.1 Data Set C-2

Analysis of data set C-2, indicates a monotonic, almost linear relationship between RSD and lot size. The analysis is summarized in Table C-1 and shown graphically in Figure C-3. The effect of larger variability on the mean value required for the plant not to exceed the standard of 1.2 lb SO_2/MM Btu is also shown in Figure C-4, which points to the plight of smaller plants (which consume less coal in a 3-hour period) as they attempt to meet the standard.

Table C-1. ANALYSIS OF DATA SET C-2[a]

Lot Size, tons			Sulfur/SO_2		
No. of data points	Range, tons	Mean, tons	Mean (%/lb/MM Btu)	Standard deviation (%/lb/MM Btu)	RSD, %
40	278-1590	1174	0.8	0.096	11.8
			1.2	0.134	10.7
40	1617-2275	1970	0.8	0.086	10.7
			1.2	0.120	9.6
40	2282-3009	2604	0.8	0.067	8.6
			1.2	0.100	8.2
39	3024-7450	4089	0.8	0.055	7.0
			1.2	0.077	6.3

[a] See Table 3-1, Data Set C-2, for comments.

C.5.2 Data For Some Other Mines

Figure C-5 depicts data for three mines.

- Mine 4186 - Leatherwood, E. Kentucky.
- Mine 8150 - Velva, North Dakota.
- Mine 1160 - Imboden, Appalachia, Virginia.

The data for all three mines refer to raw low sulfur coal. All samples were collected during the unloading process at the same time of delivery to the installation. Although the three sets of data look alike, their analyses indicate widely differing RSD's. Each has a slope essentially equal to zero, suggesting independence of RSD's and lot sizes.

Figure C-6 depicts data for two other mines:

- Mine 4830 - Thick Tiller, Dante, Virginia.
- Mine 5717 - High Splint, Pardee, Virginia

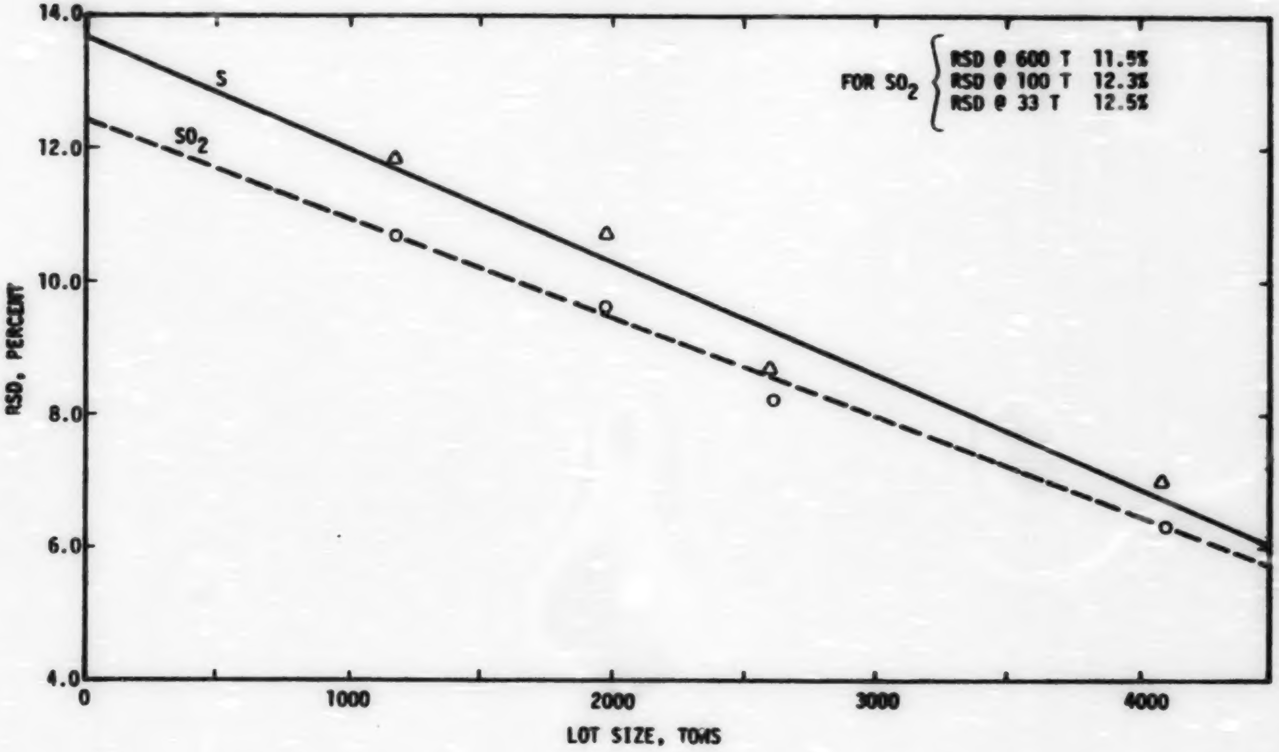

Figure C-3. Data set C-2: RSD vs. lot size tons.

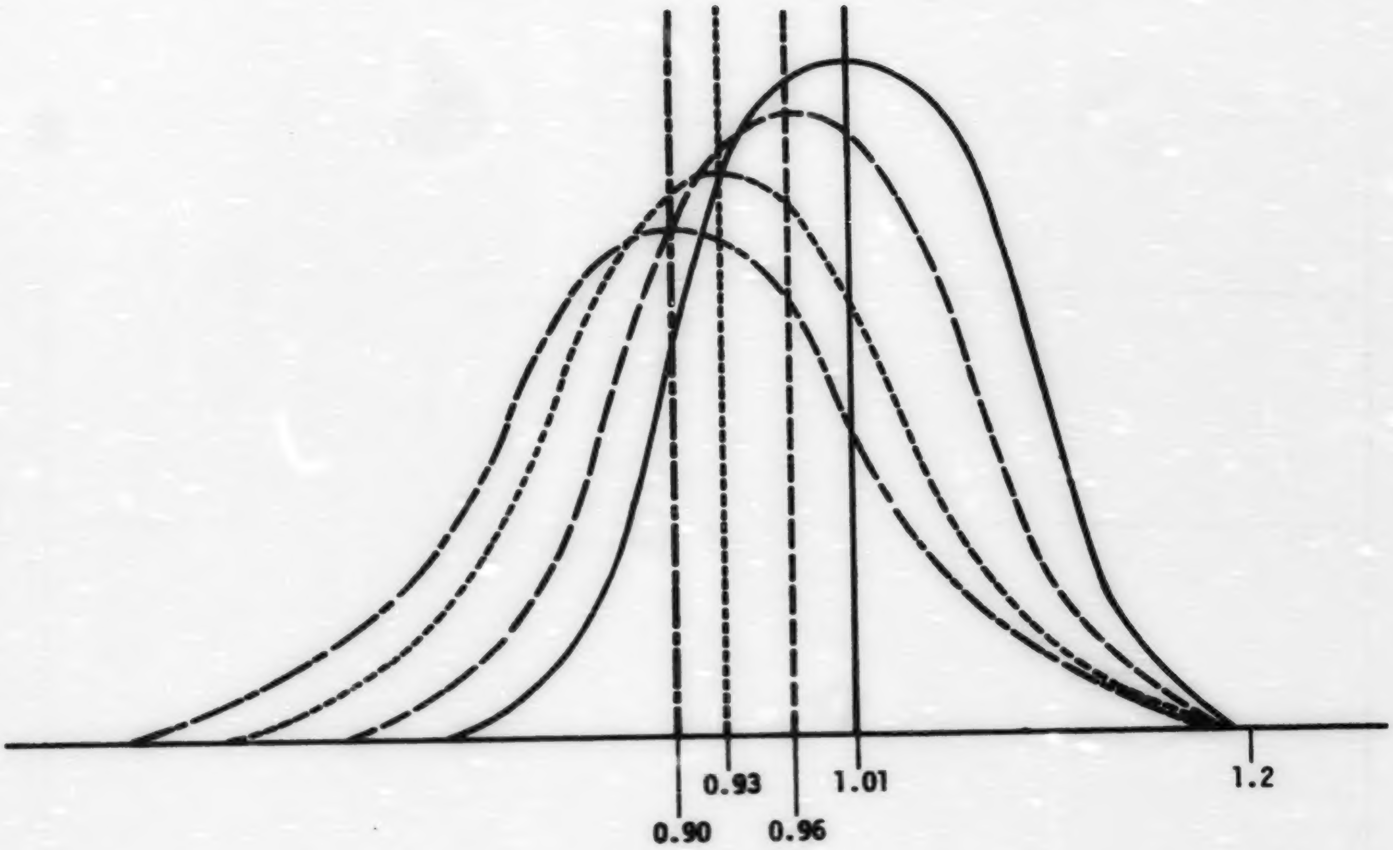

Figure C-4. Data set C-2: variability effect on the mean SO_2.

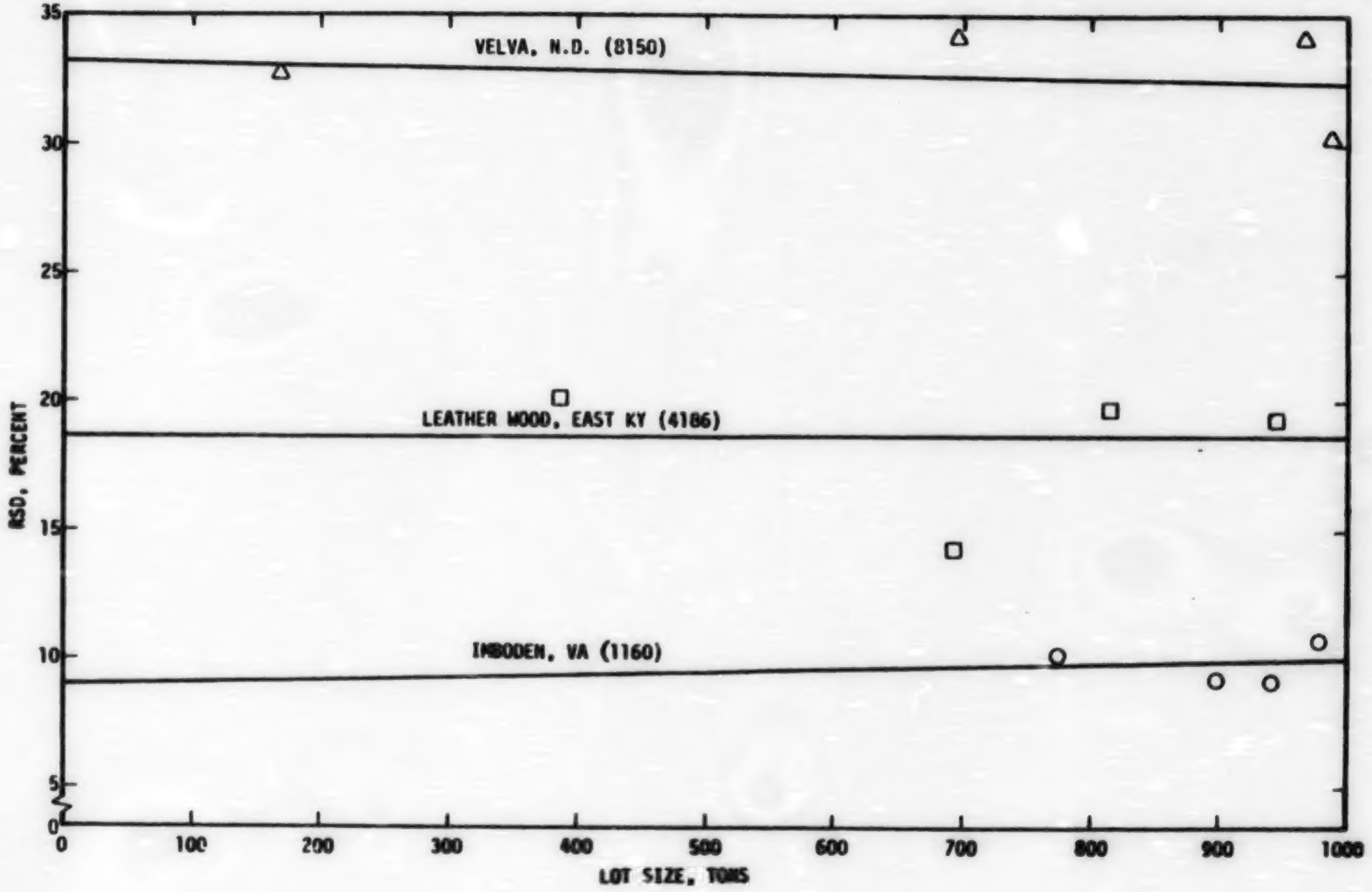

Figure C-5. RSD of SO_2 vs. lot size for mines 1160, 4186, and 8150.

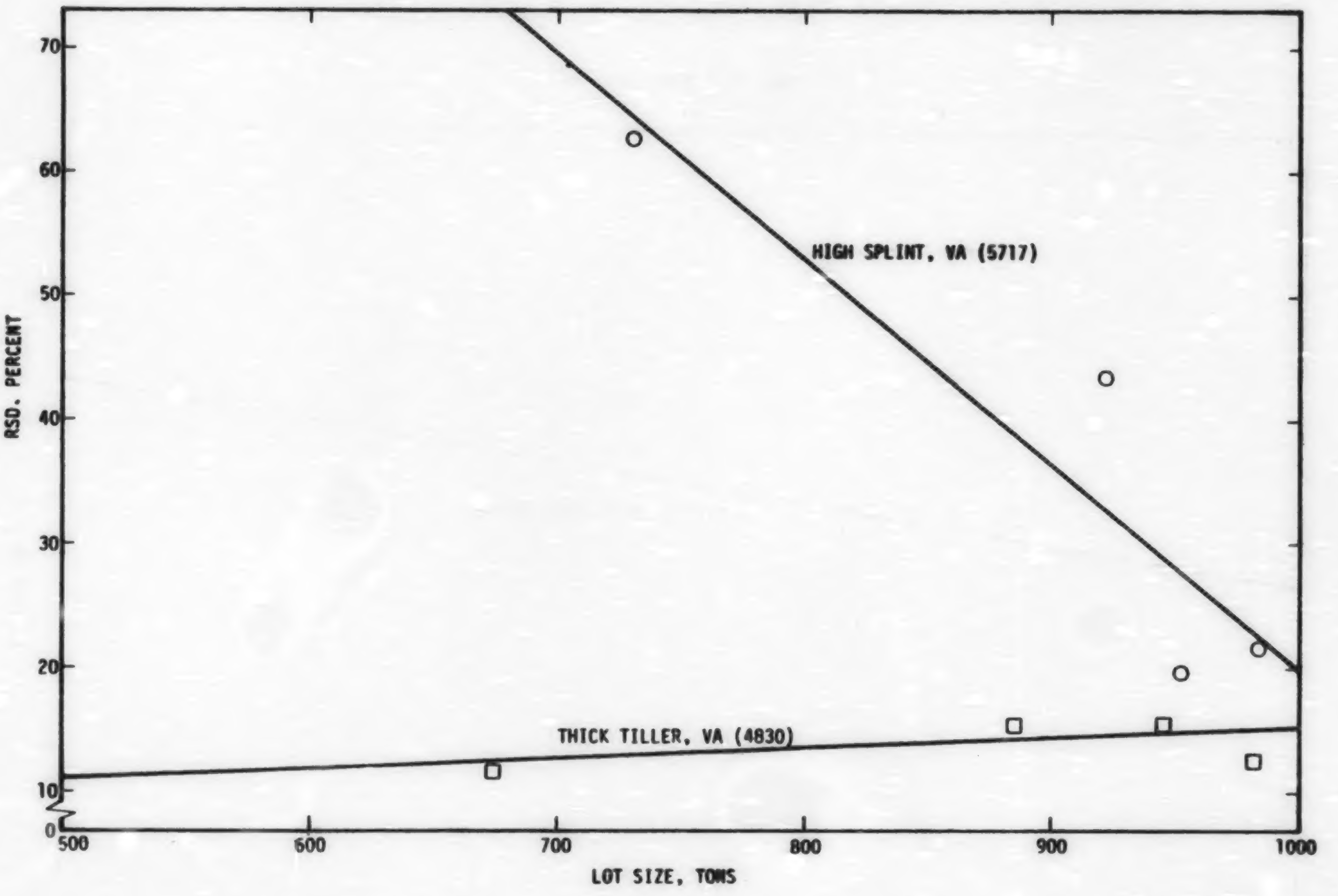

Figure C-6. RSD of SO_2 vs. lot size for mines 4830 and 5717.

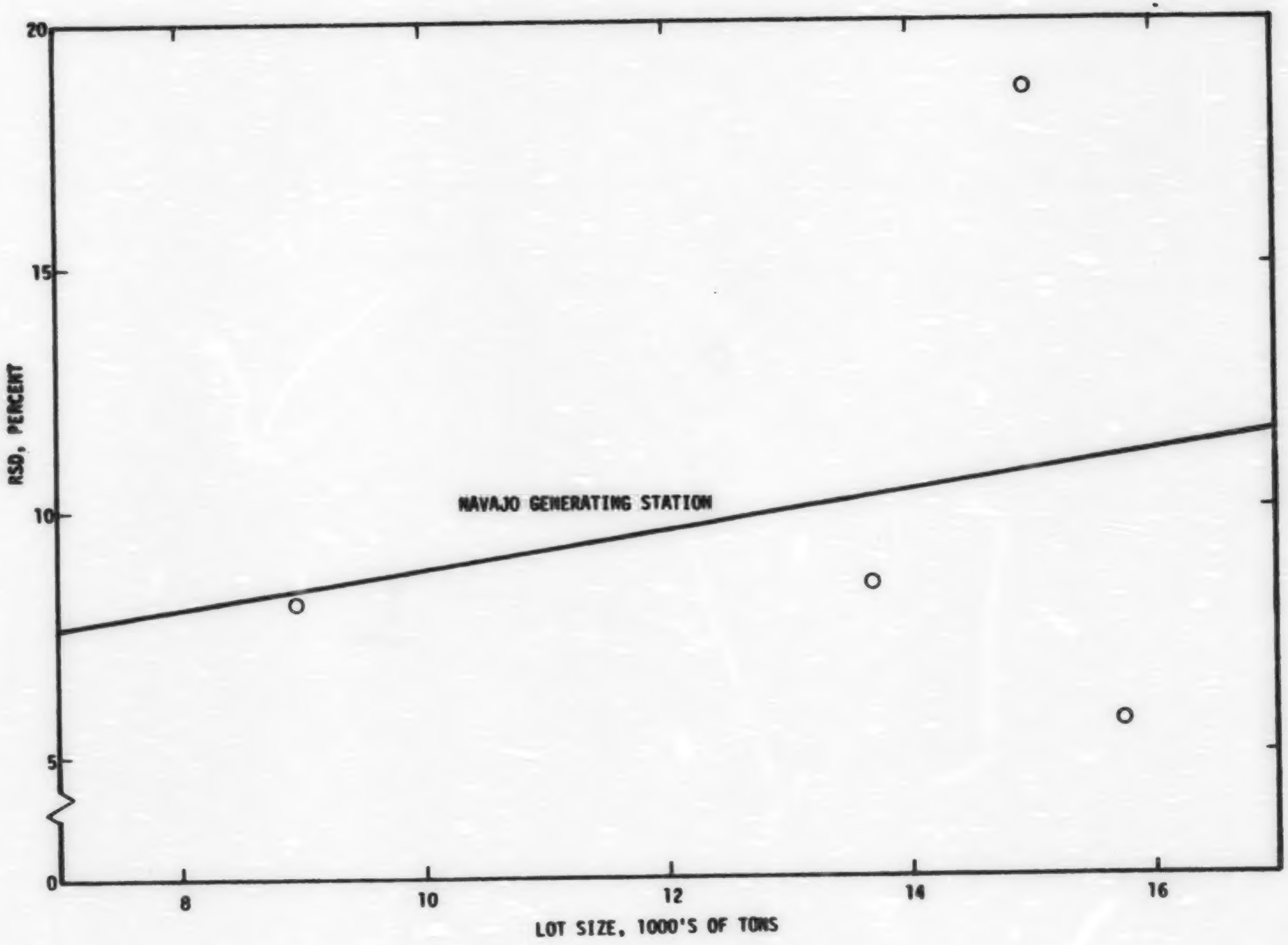

Figure C-7. RSD of SO_2 vs. lot size for NGS data.

Both these mines also yield low-sulfur coals, sampled at the time of delivery. Data for Mine 5717 (raw coal) display a rather steep downward slope, whereas data for Mine 4830 (washed coal) display a mildly upward sloping line.

Figure C-7 depicts results of an analysis of the NGS (Unit 1) data for the period 10/1/74 to 1/8/75. The plot shows local aberration in the data, causing the linear regression line to slope upward. If one neglects the aberrant point, the regression line would slope mildly downward.

C.6 CONCLUDING COMMENTS

The foregoing discussion delineates the relationship between lot size and RSD. In some cases, however, the data display either horizontal slope or upward slope with increasing lot size, defying theoretical arguments. Such cases are explained either by aberrations in data or by the fact that samples were not truly representative of the entire lot. Analyses of several other data sets, not included here, confirm at least qualitatively that RSD decreases with increasing lot size.

This discussion also brings into sharp focus the relatively greater impact of the emission standard on smaller plants and the effects of averaging over different quantities of coal or over different time periods. Consider, for example, data set C-2. Computations in this report indicate a mean of 0.93 and standard deviations for monthly averages over 600-ton lots and over 33-ton lots, by pro-rating as follows:

Monthly	$S_1 = 0.666 \times 0.070/0.153 = 0.034$	RSD = 3.62%.
Unit train	$S_2 = 0.066$	RSD = 7.10%.
500 ton	$S_3 = 0.066 \times 0.194/0.153 = 0.084$	RSD = 9.00%.
33 ton	$S_4 = 0.066 \times 0.237/0.153 = 0.102$	RSD = 11.00%.

We can now study the spread of the data by computing the $\mu + 3\sigma$ limits for $\mu = 0.93$ and for various values of standard deviations:

Monthly $L_1 = 0.93 + 3 \times 0.034 = 1.032$.

Unit train $L_2 = 0.93 + 3 \times 0.066 = 1.128$.

600 ton $L_3 = 0.93 + 3 \times 0.084 = 1.182$.

33 ton $L_4 = 0.93 + 3 \times 0.102 = 1.236$.

These limits are shown superimposed on a plot of lb SO_2/MM Btu versus the Serial No. of unit trains in Figure C-8. These limit lines indicate the extent of probable maximum fluctuations of the averages for various times/tonnages. If the standard is set at 1.2 lb SO_2/MM Btu, we can draw the following conclusion:

1. The 33-ton/3-hour plant would be in violation of the standard.
2. The 600-ton/3-hour plant would be generally in compliance (with a confidence limit of a little over 99.87 percent).
3. It would be easier to achieve compliance with longer averaging times/tonnages (notice the difference between L_1 and L_2).

Alternatively, beginning with a standard of 1.2 lb SO_2/MM Btu and RSD's as computed, we could compute the monthly average (μ_1), unit train average (μ_2), 600-ton average (μ_3), and 33-ton average (μ_4) required to meet the standard:

Monthly $\mu_1 + 0.0366 \times 3 \times \mu_1 = 1.2 \rightarrow \mu_1 = 1.08$.

Unit train $\mu_2 + 0.0710 \times 3 \times \mu_2 = 1.2 \rightarrow \mu_2 = 0.99$.

600-ton $\mu_3 + 0.0900 \times 3 \times \mu_3 = 1.2 \rightarrow \mu_3 = 0.94$.

33-ton $\mu_4 + 0.1100 \times 3 \times \mu_4 = 1.2 \rightarrow \mu_4 = 0.90$.

These calculations point to the same conclusion reached earlier.

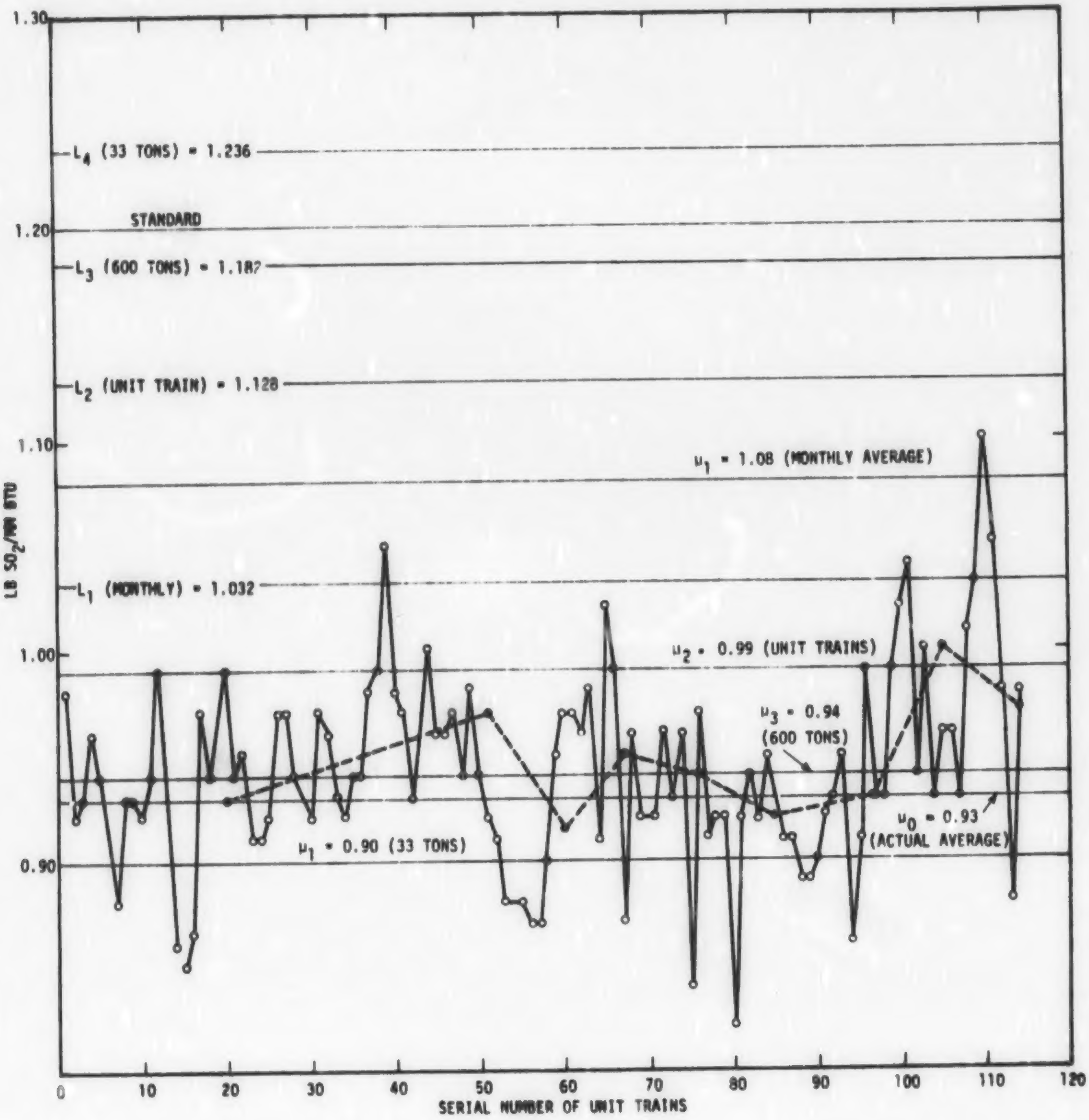

Figure C-8. Unit train and monthly variation of $SO_2/10^6$ Btu for data set C-2.

APPENDIX D
SELECTED REVIEW COMMENTS

Some of the comments of persons reviewing early drafts of this report deal with application of the approach presented herein to situations that arise in the mining/distribution/use of diverse coal resources in the United States. To the extent that these comments deal with real-world problems and provide insight into the applicability of the statistical procedures, they are considered to be of potential value to users of this report and are reproduced verbatim in this appendix.

1. "The basic problem is to determine whether the coal will be in compliance with applicable standards before the coal is mined and shipped, in fact before the coal is sold or the coal mine capital is committed for new mines or expansion of present mines. There are two possible approaches, both of which lack the desirable precision to base multi-million dollar projects.

   a. If RSD is unknown, then a value must be assumed... The tables in the report can be used for this purpose.

   b. Try to estimate the RSD. After all, the values in the report for RSD's are only averages based on various (available) sets of data. From core data and the mine plan, it may be possible to estimate (?) the RSD through simulation techniques."

2. "Item # 1 in the recommendations, page 7-3, suggests that it 'would be desirable to assess the variability of sulfur content before and after cleaning'. Also, on page 2-1, it is stated that 'the concentration of organically bound sulfur in a particular coal seam is usually lower than the

concentration of inorganic or pyritic sulfur'. The second statement has a direct bearing on the first statement in that the major portion of the sulfur removed by processes that are presently utilized by the coal industry is the inorganic or pyritic sulfur. But, in the case of many of our 'low sulfur' western reserves, this does not hold true. The pyritic sulfur is extremely low or nonexistent and, therefore, little or no reduction in the total sulfur content is shown by present economical beneficiation processes. Therefore, cleaning of these coals by present economical processes will show little if any promise of bringing these coals into compliance with the regulations as presently written. On a raw basis many of these reserves are 'marginal' as regards the 1.2 lb SO_2/MM Btu (standard) even when they are considered on a total reserve basis and 'unacceptable' a large percentage of the time when viewed on a per shipment basis of 10,000 tons."

3. "The sampling/analytical aspect is another critical point. Part 26 of the 1975 ASTM book has the following reproducibility statement for sulfur analysis: 16.2 Reproducibility - The means of results of duplicate determinations carried out by different laboratories on representative samples taken from the same bulk sample after the last stage of reduction should not differ by more than the following:

Coal containing less than the 2% sulfur	0.10%
Coal containing 2% sulfur or more	0.20%

Using these reproducibility figures the following table demonstrates the spreads that could be expected on an individual sample basis between two laboratories analyzing a split of the same sample.

lb SO_2/MM Btu using ASTM reproducibility figures

Btu	Sulfur less than 2%	Sulfur greater than 2%
12000	0.16 lb SO_2/MM Btu	0.33 lb SO_2/MM Btu
11500	0.17 lb SO_2/MM Btu	0.35 lb SO_2/MM Btu
11000	0.18 lb SO_2/MM Btu	0.36 lb SO_2/MM Btu
10500	0.19 lb SO_2/MM Btu	0.38 lb SO_2/MM Btu
10000	0.20 lb SO_2/MM Btu	0.40 lb SO_2/MM Btu
9500	0.20 lb SO_2/MM Btu	0.42 lb SO_2/MM Btu
9000	0.22 lb SO_2/MM Btu	0.44 lb SO_2/MM Btu
8500	0.24 lb SO_2/MM Btu	0.47 lb SO_2/MM Btu
8000	0.25 lb SO_2/MM Btu	0.50 lb SO_2/MM Btu

As you can see these differences do get quite large and this does not take into consideration any error that could be introduced in the sampling procedure and there are no tolerances listed for sampling."

4. "Assuming that there is no bias in the sampling/analytical procedures the reproducibility figures of an average of a series of analytical results tend to reach zero. This, in my estimation, is another valid point for the inclusion of an averaging period for SO_2 emission regulations."

5. "The procedures and examples for determining the required average sulfur content of coal to be in compliance with emission standards, as discussed in Sections 5 and 6 of the report, are based on simple cases in which all of the coal utilized by a source is obtained from one supplier and one mine. When a coal supply is obtained from twenty or more vendors, many of which use more than one mine, each with its own average sulfur content and variability, the problem of specifying and enforcing an average sulfur content becomes much more complicated. Additional problems are introduced if some of the various shipments of coal are blended to a greater or lesser degree in a coal storage pile for use at an undetermined time in the future while other shipments go

directly from the unloading facilities to the bunkers for immediate consumption. Such situations do exist in the real world today."

6. "Because the problems of procuring a complying fuel supply vary from source to source, any regulatory agency which wishes to specify the manner in which fuel analyses are to be used to demonstrate compliance with NSPS should word its regulations in such a way as to allow the source owner to select the procurement policies which will allow compliance in the most efficient manner. Such a regulation might specify that the sulfur content of the fuel, when averaged over a specified time period, must not exceed the value which will allow compliance with NSPS, more than a specified percentage of the time. This is the type of regulation discussed in example 5 on page 5-11 of the report (the example is of a state agency which requires that monthly averages of sulfur content must comply with NSPS regulation 99 percent of the time). This allows the owner of the source to specify to his/her suppliers an average suflur content which is less than that required for compliance with NSPS, or to put a portion of the burden for determining the required average sufur content on the supplier by specifying a maximum sulfur content, with a penalty to be paid by the supplier when the sulfur content exceeds the specified maximum."

7. "In a regulation of the type discussed in the preceding paragraph, the averaging time should be a period longer than a week (such as a month) because railroad scheduling problems could make it very difficult to ensure that weekly receipts from various suppliers with various sulfur contents would properly balance out."

8. "As discussed in my preliminary comments of October 22, 1976, it may not be accurate to base compliance calculations on the quality of the coal as received, in those cases where the source utilizes pulverizers which reject a portion of the pyritic suflur content of the fuel. In those cases, the sulfur content of the fuel as burned is less than the sulfur content as received; the type of pulverizer functions almost as a coal cleaning facility. The efficiency of this 'coal cleaning facility' is low when compared with conventional coal washing plants and varies with the pyritic sulfur content of the coal, but in some cases it is significant. "